

Do $1^- c\bar{c}g$ hybrid mesons exist, do they mix with charmonium ?

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March 31, 1998

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Abstract

Hybrid $c\bar{c}g$ of quantum numbers 1^{--} are considered in a quark model with constituent quarks and gluon. The lowest $J^P = 1^-$ states may be built in two ways, $l_g = 1$ (gluon excited) corresponding to an angular momentum between the gluon and the $c\bar{c}$ system, while $l_{c\bar{c}} = 1$ (quark excited) corresponds to an angular momentum between the c and the \bar{c} . The lowest lying hybrid $J^P = 1^-$ state in the flux tube model is similar to the $l_g = 1$ in the quark-gluon model. In particular it verifies the selection rule that it cannot decay into two fundamental mesons. The $l_{q\bar{q}} = 1$ hybrid may decay into two fundamental mesons, but with decay widths larger than 1 GeV, which tells that they do not really exist as resonant states. Using a chromoharmonic potential, we find no mixing between the $l_g = 1$ and $l_{c\bar{c}} = 1$. More realistic potentials might induce a strong mixing between them, implying that no hybrid meson exist. If, on the contrary, such a strong mixing does not occur, we find, in agreement with the flux tube model, that only the $l_g = 1$ appears as a real resonant state. In such a case, hybrid mesons may exist as resonances only if they are decoupled from the ground state channels, which explains the difficulty to observe them experimentally. We reconsider accordingly the Ono-Close-Page scenario of mixing between charmonium and charmed-hybrid to explain the anomalies around 4.1 GeV. We find a very small mixing between radially excited charmonium and hybrid mesons, which forbids considering the $\psi(4.040)$ and $\psi(4.160)$ as combinations of 3S charmonium and $l_g = 1$ hybrid meson with a large mixing.

LPT Oran 10/97
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hep-ph/9803470

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1 Introduction

Beyond the standard meson and baryon states, QCD predicts the existence of “exotic” color singlet states built up of constituent gluons or quarks and gluons. Checking QCD commands to search for exotics. However exotic hunting has proven along the years to be a very difficult task. Some good glueball candidates exist, such as the celebrated $f_0(1500)$ [1] but it seems very difficult to eliminate all doubts.

This is why the search for exotic non- $q\bar{q}$ quantum numbers such as 1^{-+} is so important. An undisputable experimental discovery would mean a real breakthrough. Such a state would be a very good $q\bar{q}g$ candidate. Experimental candidates exist [2], a 1^{-+} activity really seems present in $(\eta\pi)_P$ and $(\eta'\pi)_P$ channels [3], but the presence of a real resonance is still questionable.

The charmonium mass spectrum presents a noticeable anomaly in the region of the $\psi(4.040)$ and the $\psi(4.160)$. In short, the $\psi(4.040)$ is a good candidate for a $3S$ charmonium state, but then the $\psi(4.160)$ is too close to be a good $4S$ candidate, and its decay width $\Gamma(\psi(4.160) \rightarrow e^+e^-) \simeq \Gamma(\psi(4.040) \rightarrow e^+e^-)$ makes of it a bad $2D$ candidate. The anomaly has been tentatively explained by Ono [4] and more recently by Close and Page [5] as due to a mixing of the genuine $3S$ charmonium $c\bar{c}$ states with a $c\bar{c}g$ hybrid. We shall refer to this proposal as the OCP scenario. According to these authors a 1^{--} hidden-charm hybrid should exist in this mass range, and the mixing angle should be large.

The OCP scenario makes use of a selection rule forbidding the decay of (some) hybrids into ground state mesons. It results that a 1^{--} hidden-charm hybrid of mass $\simeq 4.100$ GeV would be stable if mixing with charmonium did not occur. The latter selection rule was proven in a quark model with constituent gluon [6, 7, 8, 9] for the hybrid with an orbitally excited gluon (sometimes called “transverse electric”). This selection rule was later confirmed in the flux tube model [10, 11]. The quark model also contains a state in which the gluon is in an S-wave but the c and \bar{c} quarks are in a relative P-wave. The latter quark-excited hybrids do not present the above-mentioned selection rule.

Indeed the experimental search for hybrids needs some theoretical understanding of the expected signatures of such a state. Unhappily, no QCD-based method can be operative in such a complex phenomenon. Lattice QCD may say a word about hybrid masses [12], but certainly not about decay. To our knowledge, besides some old work using QCD sum rules [13], the flux tube model and the quark model with constituent gluon are the two main tools used to study hybrid mass spectrum and decay.

The relations between these two models deserves some theoretical study. We will try to initiate this. Since we need to have some idea about the dynamics we will solve an oversimplified chromo-harmonic Hamiltonian [14]. This will also provide us with the wave functions of the hybrids and of the final mesons.

We will then compute the decay widths of both type of hybrids into ground state mesons according to the quark-gluon constituent model [6, 7, 8, 9]. The gluon-excited one does not decay as recalled above. The width of the quark-excited one will turn out to be exceedingly large. This excludes the latter for the OCP scenario. The consequences will be discussed.

The OCP scenario could then survive with the gluon-excited hybrid *if the transition*

matrix elements between $3S\ c\bar{c}$ and the hybrid allow for a large mixing angle. Unhappily the mixing will turn out much too small.

In section 2, we present and solve the model: spectrum and decay widths. In section 3 we compute the $1^{--}c\bar{c}g$ hybrid decay into ground state mesons and derive the selection rule for the gluon-excited one. In section 4 we compute the mixing of the $1^{--}c\bar{c}g$ hybrids with the $3S\ c\bar{c}$ charmonium state. We then conclude.

2 The Quark Model with constituent gluon

We shall now describe the Hamiltonian, the wave functions and the decay of hybrid mesons. The hybrid meson theoretical study is still in its infancy, although it started some time ago, presumably because no such meson has yet been unambiguously observed. It is therefore premature to recourse to sophisticated tools which anyhow would demand a number of arbitrary inputs. We need some robust method able to answer to a series of questions: spectrum, decay, mixing, etc. We choose the non-relativistic constituent quark model, well known to be semi-quantitatively successful in describing the hadron spectrum, and will compare its prediction to the more extensively used flux tube model. To make our life even simpler we will use a chromo-harmonic potential very easy to diagonalize. We hope to catch by this method the gross features of hybrid physics. This extreme simplification may nevertheless introduce some artifact as we shall discuss later on.

2.1 A simple chromo-harmonic Hamiltonian and the wave functions of the hybrid and the final mesons

We start from the Hamiltonian:

$$H = \frac{p_q^2}{2m_q} + \frac{p_{\bar{q}}^2}{2m_{\bar{q}}} + \frac{p_g^2}{2m_g} + V(r_q, r_{\bar{q}}, r_g) \quad (1)$$

where :

$$V(r_q, r_{\bar{q}}, r_g) = b_0 \sum_a \left[T_q^a T_{\bar{q}}^a (r_q - r_{\bar{q}})^2 + T_g^a T_{\bar{q}}^a (r_g - r_{\bar{q}})^2 + T_g^a T_q^a (r_g - r_q)^2 \right] \quad (2)$$

The potential (2) is spin independent (nonrelativistic quark model), it is acting on color and configuration space. It represents a matrix in color space.

We assume in our model an harmonic interaction term between the constituents which is proportional to the color matrix (T_j^a), this choice, inspired from gluon exchange in perturbative QCD, expresses the idea that the confining interaction between two constituent must be proportional to their color charge [14].

Note that :

$$\sum_a T_i^a T_j^a = \frac{1}{2} \left[(T_i^a + T_j^a)^2 - (T_i^a)^2 - (T_j^a)^2 \right] \quad (3)$$

where T^a are the generators for the SU(3) group in the different representations of the constituents, example :

$$\left(T_q^a\right)_{ij} = \frac{\lambda_{ij}^a}{2} \quad , \quad \left(T_{\bar{q}}^a\right)_{ij} = \frac{\lambda_{ij}^{a*}}{2} \quad \text{and} \quad \left(T_g^a\right)_{bc} = -if_{bc}^a = F_{bc}^a . \quad (4)$$

$i+j$ is necessarily in a well defined color representation, such that combined with the third constituent it gives an overall color singlet. Hence not only $T_i^{a^2}$ and $T_j^{a^2}$ are the identity times eigenvalues of the Casimir operator but also $(T_i^a + T_j^a)^2$ is the Casimir eigenvalue for the color representation of the third constituent.

Finally :

$$\sum_a T_i^a T_j^a = \alpha_{ij} \mathbb{1} \quad (5)$$

where $\mathbb{1}$ is identity matrix in color space and

$$\alpha_{ij} \equiv \frac{1}{2} [C(k) - C(i) - C(j)] \quad (6)$$

where k is the third constituent and $C(i)$ is the Casimir of i .

From $C(3) = 4/3$ and $C(8) = 3$ we find

$$\begin{aligned} \alpha_{q\bar{q}} &= 1/6 \\ \alpha_{qg} &= \alpha_{\bar{q}g} = -3/2 \quad . \end{aligned} \quad (7)$$

Finally the potential V becomes independent of colors indices and all information about color representation are encoded in the number α_{ij} :

$$V(r_q, r_{\bar{q}}, r_g) = b_0 \sum_{\substack{i,j=q,\bar{q},g \\ i < j}} \alpha_{ij} (r_i - r_j)^2 \quad (8)$$

From (1), (2) and (3) we obtain the Hamiltonian for color singlet $q\bar{q}g$ system :

$$H_{hyb} = \frac{P^2}{2M} + \frac{p_{q\bar{q}}^2}{2\mu_{q\bar{q}}} + \frac{k^2}{2\mu_g} - \frac{7b_0}{12} r_{q\bar{q}}^2 - 3b_0 r^2 \quad (9)$$

where $P = p_q + p_{\bar{q}} + p_g$ is the center of mass momentum of $q\bar{q}g$ system, $p_{q\bar{q}} = \frac{m_{\bar{q}}p_q - m_q p_{\bar{q}}}{m_q + m_{\bar{q}}}$

is the relative momentum between q and \bar{q} , $k = \frac{p_g(m_q + m_{\bar{q}}) - (p_q + p_{\bar{q}})m_g}{m_g(m_q + m_{\bar{q}})}$ is the relative momentum between g and $q\bar{q}$. Their conjugate variables are $r_{q\bar{q}} = r_q - r_{\bar{q}}$ and $r = r_g - \frac{r_q + r_{\bar{q}}}{2}$.

$$M = m_q + m_{\bar{q}} + m_g \quad ; \quad \mu_{q\bar{q}} = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}} \quad \text{and} \quad \mu_g = \frac{(m_q + m_{\bar{q}})m_g}{m_q + m_{\bar{q}} + m_g} . \quad (10)$$

The eigenstates of the Schrödinger equation (9) are :

$$\psi_{\ell_{q\bar{q}}}^{m_{q\bar{q}}}(p_{q\bar{q}}) = \left\{ \frac{16\pi^3 R_{q\bar{q}}^{2\ell_{q\bar{q}}+3}}{\Gamma\left(\frac{3}{2} + \ell_{q\bar{q}}\right)} \right\}^{1/2} y_{\ell_{q\bar{q}}}^{m_{q\bar{q}}}(\vec{p}_{q\bar{q}}) e^{-\frac{1}{2}(R_{q\bar{q}}^2 p_{q\bar{q}}^2)} \quad (11)$$

and

$$\psi_{\ell_g}^{m_g}(k) = \left\{ \frac{16\pi^3 R_g^{2\ell_g+3}}{\Gamma\left(\frac{3}{2} + \ell_g\right)} \right\}^{1/2} y_{\ell_g}^{m_g}(\vec{k}) e^{-\frac{1}{2}R_g^2 k^2} \quad (12)$$

where

$$y_{\ell}^m(\vec{p}) \equiv p^{\ell} Y_{\ell}^m(\theta, \Omega) \quad . \quad (13)$$

and where :

$$\begin{aligned} R_{q\bar{q}}^2 &= (2\mu_{q\bar{q}}(-7b_0/12))^{-1/2} \\ R_g^2 &= (2\mu_g(-3b_0))^{-1/2} \end{aligned} \quad (14)$$

It is noticeable that in the case of the chromoharmonic Hamiltonian (1)-(2) the quantum numbers $\ell_{q\bar{q}}$ and ℓ_g are diagonalized. This feature would not remain valid for a more general potential. This will be discussed later.

Analogously the Hamiltonian (1)-(2) restricted to a meson state writes :

$$H_{meson} = \frac{P^2}{2M} + \frac{p^2}{2\mu} - \frac{4b_0}{3} r^2 \quad (15)$$

where $P = p_q + p_{\bar{q}}$, $p = \frac{m_{\bar{q}}p_q - m_q p_{\bar{q}}}{m_q + m_{\bar{q}}}$, conjugate to $r = r_q - r_{\bar{q}}$ and where :

$$M = m_q + m_{\bar{q}} \quad ; \quad \mu = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}} \quad . \quad (16)$$

The final states mesons, which will be labelled B and C , will be described by the eigenstates of eq. (15):

$$\psi_{\ell_B}^{m_B}(p) = \left\{ \frac{16\pi^3 R_B^{2\ell_B+3}}{\Gamma\left(\frac{3}{2} + \ell_B\right)} \right\}^{1/2} y_{\ell_B}^{m_B}(\vec{p}) e^{-\frac{1}{2}R_B^2 p^2} \quad (17)$$

with :

$$R_B^2 = (2\mu_B(-4b_0/3))^{-1/2} \quad (18)$$

and the same for ψ_C .

2.2 General formulae for hybrid decay into two mesons

The Classification and the decay of hybrid mesons in a constituent model have been studied in [6, 7, 8, 9] ; we will use here the notations of [7] :

ℓ_g : is the relative orbital momentum between the gluon and the $q\bar{q}$ center of mass
 $\ell_{q\bar{q}}$: is the relative orbital momentum between q and \bar{q}

$S_{q\bar{q}}$: is the total quark spin
 j_g : is the total gluon angular momentum
 L : $\ell_{q\bar{q}} + J_g$.

The parity and charge conjugation of the hybrid are given by :

$$\begin{aligned}
 P &= (-)^{\ell_{q\bar{q}} + \ell_g} \\
 C &= (-)^{\ell_{q\bar{q}} + S_{q\bar{q}} + 1} .
 \end{aligned} \tag{19}$$

To lowest order the decay is described by the matrix element of the QCD:

$$H = g \int dx \Psi(x) \gamma_\mu \frac{\lambda^a}{2} \psi(x) A_a^\mu(x) \tag{20}$$

we expand at $t = 0$

$$\psi(x) = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\vec{x}} \left(u_{ps} b_{ps} + v_{ps} d_{ps}^+ \right) \tag{21}$$

$$A_a^\mu(x) = \sum_{\lambda=1}^2 \int \frac{d^3k}{\sqrt{2\omega}(2\pi)^3} \varepsilon_{\kappa\lambda a}^\mu \left(a_{\kappa\lambda a} e^{i\vec{k}\vec{x}} + a_{\kappa\lambda a}^+ e^{-i\vec{k}\vec{x}} \right) \tag{22}$$

with φ_a being color representation and :

$$\begin{aligned}
 \{ b_{ps\rho}, b_{p's'\rho'}^+ \} &= (2\pi)^3 \delta^3(p - p') \delta_{s,s'} \delta_{\rho,\rho'} \\
 [a_{\kappa\lambda a}, a_{\kappa'\lambda'a'}^+] &= (2\pi)^3 \delta^3(k - k') \delta_{\lambda,\lambda'} \delta_{a,a'} .
 \end{aligned} \tag{23}$$

The hamiltonian annihilating a gluon and creating a quark pair is :

$$H = g \sum_{ss'\lambda\zeta\zeta'} \int \frac{d^3p}{\sqrt{2\omega}(2\pi)^9} \frac{d^3k}{\sqrt{2\omega}(2\pi)^9} \frac{d^3p'}{\sqrt{2\omega}(2\pi)^9} (2\pi)^3 \delta_3(p - p' - k) \left(\bar{u}_{ps} \gamma_\mu \frac{\lambda_{\zeta\zeta'}^a}{2} v_{p's'} \right) b_{ps\zeta}^+ d_{p's'\zeta'}^+ \varepsilon_{\kappa\lambda}^\mu a_{\kappa\lambda a} \tag{24}$$

in the nonrelativistic limit :

$$\bar{u}_{ps} \gamma_\mu v_{p's'} \varepsilon_{\kappa\lambda}^\mu = \chi_s^+ \vec{\sigma} \chi_{s'} \vec{\varepsilon}_{\kappa\lambda} \tag{25}$$

where $\chi_{s'}$ is the antiquark spinor in the complex conjugate representation.

The meson state and the hybrid state are given respectively in the nonrelativistic approximation by :

$$\begin{aligned}
 |B, M_B \rangle &= \sum_{\substack{ss' \\ a m \mu}} \int \frac{d^3p_q}{(2\pi)^6} \frac{d^3p_{\bar{q}}}{\sqrt{3}} \chi_{ss'}^{\mu_B} (2\pi)^3 \delta_3(p_B - p_q - p_{\bar{q}}) \\
 \psi_{\ell_B}^{m_B} \left(\frac{m_{\bar{q}} p_q - m_q p_{\bar{q}}}{m_q + m_{\bar{q}}} \right) &< \ell_B m_B S_B \mu_B | J_B M_B \rangle b_{p_q s a}^+ d_{p_{\bar{q}} s' a}^+ | O \rangle .
 \end{aligned} \tag{26}$$

with $\int d^3p/(2\pi)^3 |\psi_B(p)|^2 = 1$ and analogously for the C .

Note here that q and \bar{q} can be different in flavour. The hybrid meson state in its rest frame is given by

$$\begin{aligned}
|A, L, M \rangle &= \sum \int \frac{d^3p_q d^3p_{\bar{q}} d^3k}{(2\pi)^9} \chi_{s_q s_{\bar{q}}}^{\mu_{q\bar{q}}} (2\pi)^3 \delta_3(k + p_q + p_{\bar{q}}) \frac{\lambda_{c_q c_{\bar{q}}}^{c_g}}{4} \\
&\psi_{\ell_{q\bar{q}}}^{m_{q\bar{q}}} \left(\frac{m_{\bar{q}} p_q - m_q p_{\bar{q}}}{m_q + m_{\bar{q}}} \right) \psi_{\ell_g}^{m_g} \left(\frac{(m_q + m_{\bar{q}})k - m_g(p_q + p_{\bar{q}})}{m_q + m_{\bar{q}} + m_g} \right) \\
&\langle \ell_g m_g 1 \mu_g | J_g M_g \rangle \langle \ell_{q\bar{q}} m_{q\bar{q}} J_g M_g | L m' \rangle \langle L m' S_{q\bar{q}} \mu_{q\bar{q}} | J M \rangle \\
&b_{p_q s_q c_q}^+ d_{p_{\bar{q}} s_{\bar{q}} c_{\bar{q}}}^+ a_{\kappa, \mu_g c_g}^+ | 0 \rangle
\end{aligned} \tag{27}$$

where c_q , $c_{\bar{q}}$ and c_g are the color charge of the quark, antiquark and gluon with $c_q = 1, \dots, 3$; $c_{\bar{q}} = 1, \dots, 3$; $c_g = 1, \dots, 8$ and a sum over repeated indices is understood. Using (24) one gets in a straightforward manner the matrix element $\langle B, M_B C, M_C | H | A, L, M \rangle$ between an hybrid state A and two mesons B and C :

$$\langle B, M_B C, M_C | H | A, L, M \rangle = g f(A, B, C) (2\pi)^3 \delta_3(p_A - p_B - p_C) \tag{28}$$

where $f(A, B, C)$ representing the decay amplitude by :

$$\begin{aligned}
f(A, B, C) &= \sum_{\substack{m_{q\bar{q}}, m_g, m_B, m_C \\ \mu_{q\bar{q}}, \mu_g, \mu_B, \mu_C}} \Omega \phi X(\mu_{q\bar{q}}, \mu_g; \mu_B, \mu_C) \\
&I(m_{q\bar{q}}, m_g; m_B, m_C, m) \langle \ell_g m_g 1 \mu_g | J_g M_g \rangle \\
&\langle \ell_{q\bar{q}} m_{q\bar{q}} J_g M_g | L m' \rangle \langle L m' S_{q\bar{q}} \mu_{q\bar{q}} | J M \rangle \\
&\langle \ell_B m_B S_B \mu_B | J_B M_B \rangle \langle \ell_C m_C S_C \mu_C | J_C M_C \rangle
\end{aligned} \tag{29}$$

where Ω , X , I and ϕ are the color, spin, spatial and flavour overlaps. Ω is given by :

$$\Omega = \frac{1}{24} \sum_a \text{Tr} (\lambda^a)^2 = \frac{2}{3} \tag{30}$$

from :

$$\chi_{\mu_1}^+ \sigma^\lambda \chi_{\mu_2} = \sqrt{3} \langle \frac{1}{2} \mu_2 1 \lambda | \frac{1}{2} \mu_1 \rangle \tag{31}$$

where $\sigma^{\pm 1} \equiv \mp \frac{1}{\sqrt{2}} (\sigma_x \pm i \sigma_y)$ and $\sigma^0 = \sigma_z$, we obtain the spin overlap :

$$\begin{aligned}
X(\mu_{q\bar{q}}, \mu_g; \mu_B, \mu_C) &= \sum_s \sqrt{2} \begin{bmatrix} 1/2 & 1/2 & S_B \\ 1/2 & 1/2 & S_C \\ S_{q\bar{q}} & 1 & S \end{bmatrix} \langle S_{q\bar{q}} \mu_{q\bar{q}} 1 \mu_g | S \mu_{q\bar{q}} + \mu_g \rangle \\
&\langle S_B \mu_B S_C \mu_C | S \mu_B + \mu_C \rangle
\end{aligned} \tag{32}$$

where :

$$\begin{bmatrix} 1/2 & 1/2 & S_B \\ 1/2 & 1/2 & S_C \\ S_{q\bar{q}} & 1 & S \end{bmatrix} = \sqrt{(2S_B + 1)(2S_C + 1)3(2S_{q\bar{q}} + 1)} \begin{Bmatrix} 1/2 & 1/2 & S_B \\ 1/2 & 1/2 & S_C \\ S_{q\bar{q}} & 1 & S \end{Bmatrix} \quad (33)$$

the spatial overlap is given by:

$$I(m_{q\bar{q}}, m_g; m_B, m_C, m) = \int \int \frac{dp dk}{\sqrt{2\omega}(2\pi)^6} \psi_{\ell_{q\bar{q}}}^{m_{q\bar{q}}}(p_B - p) \psi_{\ell_g}^{m_g}(k) \quad (34)$$

$$\psi_{\ell_B}^{m_B^*} \left(\frac{p_B m_{\bar{q}_i}}{m_q + m_{\bar{q}_i}} - p - \frac{k}{2} \right) \psi_{\ell_C}^{m_C^*} \left(-\frac{m_{q_i} p_B}{m_{q_i} + m_{\bar{q}}} + p - \frac{k}{2} \right) d\Omega_B Y_{\ell}^{m^*}(\Omega_B)$$

The $Y_{\ell}^{m^*}(\Omega_B)$ spherical harmonic, integrated over the 4π solid angle for Ω_B projects out the decay amplitude on the orbital momentum ℓ, m between the two final mesons. The $q_i \bar{q}_i$ form the created quark pair.

Finally:

$$\phi = \begin{bmatrix} i_1 & i_3 & I_B \\ i_2 & i_4 & I_C \\ I_A & 0 & I_A \end{bmatrix} \eta \varepsilon \quad (35)$$

where the I 's (i 's) label the hadron (quark) isospins, $\eta = 1$ if the gluon goes into strange quarks and $\eta = \sqrt{2}$ if it goes into nonstrange ones. ε is the number of diagrams contributing to the decay. Indeed one can check that since P and C are conserved, two diagrams contribute with the same sign and magnitude for allowed decays while they cancel for forbidden ones. In the case of two identical final particles, $\varepsilon = \sqrt{2}$. The partial width is then given by :

$$\Gamma(A \rightarrow BC) = 4 \alpha_s |f(A, B, C)|^2 \frac{P_B E_B E_C}{M_A} \quad (36)$$

with :

$$P_B^2 = \frac{[M_A^2 - (m_B + m_C)^2][M_A^2 - (m_B - m_C)^2]}{4M_A^2} \quad ;$$

$$E_B = \sqrt{P_B^2 + m_B^2} \quad , \quad E_C = \sqrt{P_B^2 + m_C^2} \quad . \quad (37)$$

3 The 1^{--} hybrids decay and the selection rule

3.1 Classification of the 1^{--} hybrid states

Let us now consider the lightest $J^{PC} = 1^{--}$ hybrid mesons, using the quark model with a constituent gluon [6, 7, 8, 9] as developed in section 2. Eq. (19) implies $\ell_{q\bar{q}} = S_{q\bar{q}}$ and

$\ell_{q\bar{q}} + l_g$ odd. The lightest such states are $\ell_{q\bar{q}} = S_{q\bar{q}} = 0, l_g = 1$, which we shall refer to as the gluon-excited hybrid, and $\ell_{q\bar{q}} = S_{q\bar{q}} = 1, l_g = 0$, which we shall refer to as the quark-excited hybrid. In the case of the gluon-excited one (respectively the quark excited one) we obtain from the Clebsch Gordan of the eq. (27) that $L = J = J_g = 1$ (respectively $L = 0, 1, 2; J_g = 1$).

P	C	ℓ_g	$\ell_{q\bar{q}}$	J_g	$S_{q\bar{q}}$	L	J
–	–	0	1	1	1	0	1
–	–	0	1	1	1	1	1
–	–	0	1	1	1	2	1
–	–	1	0	1	0	1	1

Table 1: Lowest $J^{PC} = 1^{--} q\bar{q}g$ hybrid mesons and their quantum numbers

Notice that the gluon-excited state, verifying $\ell_g = J_g = 1$, is one member of what has been sometimes called a “transverse-electric” hybrid [7], while the quark-excited states with $\ell_g \neq J_g$ are “transverse-magnetic”. We will not use this nomenclature which does not seem too convenient.

3.2 Decay into ground state mesons and the selection rule

Let us compute the integral (34) for ground $l_B = l_C = 0$ assuming $R_B = R_C$:

$$I(m_{q\bar{q}}, 0; 0, 0, m) = 2^4 \sqrt{\frac{\pi}{3\omega}} \frac{R_{q\bar{q}}^{3/2+\ell_{q\bar{q}}} R_g^{3/2+\ell_g} R_B^5}{\left(R_g^2 + R_B^2/2\right)^{3/2} \left(R_{q\bar{q}}^2 + 2R_B^2\right)^{5/2} m_q + m_{\bar{q}_i}} \frac{2m_q}{m_q + m_{\bar{q}_i}} P_B \quad (38)$$

$$\exp \left\{ -\frac{P_B^2}{2} \left[R_{q\bar{q}}^2 + \frac{2m_{\bar{q}_i}^2 R_B^2}{(m_q + m_{\bar{q}_i})^2} - \frac{\left[2m_{q_i} R_B^2 + (m_q + m_{\bar{q}_i}) R_{q\bar{q}}^2 \right]^2}{\left(R_{q\bar{q}}^2 + 2R_B^2 \right) (m_q + m_{\bar{q}_i})^2} \right] \right\} \delta_{\ell_g, 0} \delta_{\ell_{q\bar{q}}, \ell} \delta_{m_{q\bar{q}}, m}$$

the $\delta_{\ell_g, 0}$ term in the last formula tells us that a gluon-excited hybrid cannot decay into two ground state mesons. This is the well known *selection rule for the gluon-excited hybrids decay* [6, 7, 8, 10, 11]. This selection rule is a basic ingredient of the OCP scenario as will be discussed below. The $\delta_{\ell_{q\bar{q}}, \ell}$ factor shows us that the intermeson orbital momentum is a direct measure of the interquark orbital momentum in the hybrid when $R_B = R_C$.

3.3 Decay widths of quark-excited hybrids

To fix the b_0 parameter of the harmonic oscillator potential (9) we will use the level spacing of the charmonium, charmed and the light mesons with Hamiltonian (15). We present in this table the different values of b_0 :

meson sector	$\omega_{exp}(\text{GeV})$	$\mu(\text{GeV})$	$R_{exp}^2(\text{GeV}^{-2})$	$b_0(\text{GeV}^3)$
light meson	0.465	0.175	12.28	- 0.015
charmonium meson	0.295	0.85	3.98	- 0.03
charm meson	0.453	0.3	7.35	- 0.025

Table 2: b_0 fitted from meson level spacing. ω_{exp} is the roughly averaged experimental level spacing between the ground state and the first excited state and μ is the reduced mass in eq. (16).

The fact that in table 2 the value of b_0 varies from line to line reflects the well known fact that the harmonic oscillator potential is not a good potential for mesons. However, we have checked that none of our results in this paper will depend drastically on the variation of b_0 within the range -0.015 to -0.03 GeV^3 . It is why we will only present the results for one central value $b_0 = -0.02$ GeV^3 for decays widths and the mixing.

In Eq. (14) we use the following masses :

$$m_c = m_{\bar{c}} = 1.7 \text{ GeV}, \quad m_u = m_d = 0.35 \text{ GeV} \quad \text{and} \quad m_g = 0.8 \text{ GeV} \quad .$$

From (14-18) we obtain the corresponding radii with $b_0 = -0.02$ GeV^3 :

$$\begin{aligned} R_{c\bar{c}}^2 &= 7.1 \text{ GeV}^{-2} \\ R_B^2 &= 7.9 \text{ GeV}^{-2} \\ R_g^2 &= 3.58 \text{ GeV}^{-2} \quad . \end{aligned} \quad (39)$$

Remember that the level spacing is given by $\omega = 1/(\mu R^2)$.

The decays width of the hybrid Charmonium 1^{--} :

					$\Gamma_{D^{*0}\bar{D}^{*0}} = \Gamma_{D^{*-}D^{*+}}$				
$\Gamma_{D^0\bar{D}^0}$	$\Gamma_{D^+D^-}$	$\Gamma_{D_s^+D_s^-}$	$\Gamma_{D^{*0}\bar{D}^0}$	$\Gamma_{\bar{D}^{*0}D^0}$	$S = 0$	$S = 1$	$S = 2$	$\Gamma_{tot D^{*0}\bar{D}^{*0}}$	$\Gamma_{tot}(4.04)$
176	179	135	357	357	6,7	0	135	141,7	1487,1
528	536	405	268	268	20	0	101,5	121,5	2248
880	894	674	446	446	34	0	6,7	40,7	3421,1

Table 3: Predicted widths in α_s Mev for the decay of a quark-excited ($l_{c\bar{c}} = 1$) $c\bar{c}g$ 1^{--} hybrid meson of mass (4.04) GeV

The decay widths of the quark-excited hybrid charmonium are very large, much larger than the level spacing, meaning that such states do not really emerge from the continuum

					$\Gamma_{D^{*0}\bar{D}^{*0}} = \Gamma_{D^{*-}D^{*+}}$				
$\Gamma_{D^0\bar{D}^0}$	$\Gamma_{D^+D^-}$	$\Gamma_{D_s^+D_s^-}$	$\Gamma_{D^{*0}\bar{D}^0}$	$\Gamma_{\bar{D}^{*0}D^0}$	$S = 0$	$S = 1$	$S = 2$	$\Gamma_{tot D^{*0}\bar{D}^{*0}}$	$\Gamma_{tot}(4.16)$
132	136	196	372	372	57,5	0	1151	1208,5	3625
396	407,5	588	279	279	173	0	863,5	1036,5	4022,5
659	679	980	465	465	288	0	57,5	345,5	3939

Table 4: Predicted widths in α_s Mev quark-excited ($\ell_{c\bar{c}} = 1$) $c\bar{c}g1^{--}$ hybrid meson of mass (4.16) GeV

of the two meson spectrum. In other words, the quark-excited hybrids do decay before they have had time to really exist as a hadron. *The quark-excited hybrid do not exist as resonances.*

At this stage, in the chromo-harmonic model we have a very strongly cut distinction between the gluon excited ($\ell_g = 1$) hybrids which are not allowed to decay, and the quark excited ones ($\ell_{q\bar{q}} = 1$), too broad to be considered as resonances. *We may fear* that using a more realistic potential, the mixing between these two types of hybrids will produce only broad eigenstates, i.e. lead us to conclude *that maybe* 1^{--} hybrids do not really exist.

3.4 A comparative look at the flux tube model

The flux tube model [15] assumes that hybrids are predominantly quark-antiquark states moving on an adiabatic surface generated by an excited color flux tube. There is no constituent gluon as in the quark-gluon constituent model, the gluon degrees of freedom are treated as collective excitations of the color flux. In practice the color is treated as a string, and the excited states are represented by the excited modes of the string.

This picture seems to be close to the picture of a gluon orbitally excited around the center of mass. Indeed, looking at eq. (4) in [10], the wave function of the excited hybrid in the flux tube model is very similar to the one in eq. (12) in this paper³. The variable y in [10] happens to be the variable k in this paper. It's why the flux tube model predicts the same selection rule as the Quark model with constituent gluon in the gluon-excited mode.

The quark excited mode looks similar to the next to lowest lying hybrid in the flux tube model⁴ formed from a vanishing total orbital momentum in the hybrid and $S_{c\bar{c}} = 1$. Indeed, the first line in table 1 has exactly the latter quantum numbers. In the flux tube model, the state with $S_{c\bar{c}} = 1$ does not follow the selection rule for the decay into ground state mesons [11]. One difference between the two models is that in the constituent model the gluon excited mode is lighter than the quark excited, as can be seen from eq. (9): the harmonic coefficient of the gluon mode is $-3b_0$, much larger than that of the quark

³But it is totally different from the quark-excited state eq. (11).

⁴We are very much indebted to Philip Page for having drawn our attention on this similarity.

mode $-7b_0/12$, while the reduced masses are of the same order. Using the parameters of the preceding section, we find an excitation energy of ~ 200 MeV for the quark excited mode, versus ~ 600 MeV for the gluon excited one.

4 Hybrid-charmonium mixing

In the charmonium spectroscopy, the $\psi(4040)$ and $\psi(4160)$ ($J^{PC} = 1^{--}$) were traditionally believed and regarded as $3S$ and $4S$ $c\bar{c}$ respectively, but the fact that these two states are only split by about ~ 100 MeV while the normal splitting in the conventional charmonium picture is about ~ 400 MeV has lead to ask some questions about the real composition of these objects.

A suggestion to explain the anomalous narrow $\psi(4040) - \psi(4160)$ splitting is to assume the existence of an additional state: an hybrid meson with a mass around 4.1 GeV mixed with conventional $\psi(3S)$ charmonium around 4.1 GeV. The Ono-Close-Page (OCP) scenario [4], [5] propose the physical eigenstates to be :

$$\psi_{\pm} \equiv \frac{1}{\sqrt{2}} (\psi(3S) \pm H_C) \quad (40)$$

which they identify as : $\psi_- \equiv \psi(4040)$ and $\psi_+ \equiv \psi(4160)$, H_C is an hybrid state. They use the selection rule for hybrid decay which, in the flux tube model, holds for the lowest lying hybrid as already mentioned [10, 11].

In order to study the possibility for the OCP scenario in the present model we will compute the mixing pattern of the conventional $\psi(3S)$ with the hybrid charmonium $c\bar{c}g, 1^{--}$ mesons, listed in table 1. From the results about the decay widths, only the gluon-excited one, $l_g = 1$ might fit into the OCP scenario, since, in the absence of mixing with the quark-excited ones it verifies the same decay selection rule as in the flux-tube model. For the sake of completeness, we also compute the mixing with the quark-excited hybrids.

4.1 Hybrid-charmonium transition Hamiltonian

The transition Hamiltonian is given by QCD :

$$H_{trans} = g \int dx \bar{\psi}(x) \gamma_{\mu} \frac{\lambda^a}{2} \psi(x) A_a^{\mu}(x) \quad (41)$$

from (21), (22) and (23) we express the Hamiltonian in Fock space and expand to the first non vanishing order in v/c (i.e. the first relativistic correction) and get $H_{trans} = H_1 + H_2$ where H_1 (H_2) is the Hamiltonian in which the quark (antiquark) emits a gluon, with

$$H_1 = g \sum_{\substack{\lambda\lambda'\bar{\lambda}\bar{\lambda}' \\ ab'\bar{b}\bar{b}'}} \int \frac{d^3p_c d^3p_{c'} d^3k}{(2\pi)^9 \sqrt{2\omega}} (2\pi)^3 \delta(p_c - k - p_{c'}) b_{p_c\lambda b}^+ b_{p_{c'}\lambda' b'}^+ \frac{\lambda_{b'b}^a}{2} a_{k\mu g}^+ \\ \delta_{\bar{b},\bar{b}'} \delta_{\bar{\lambda},\bar{\lambda}'} \chi_{\lambda'}^+ \left[\frac{\vec{\varepsilon}^{(\mu_g)}(\vec{p}_c + \vec{p}_{c'})}{2m} + \frac{\vec{\varepsilon}^{(\mu_g)}\vec{k}}{m_g} + \frac{i\vec{\sigma}}{2m} (\vec{\varepsilon}^{(\mu_g)} \wedge \vec{k}) \right] \chi_{\lambda} dp_{\bar{c}} \delta_3(p_{\bar{c}} - p_{\bar{c}'}) \quad (42)$$

where $\lambda\lambda'$ are the quark spin properties, b, b' the quark color indices and a labels the gluon colors, $\vec{\varepsilon}^{(\mu_g)}$ the gluon polarisation and :

$$H_2 = -g \sum_{\substack{\lambda\lambda'\bar{\lambda}\bar{\lambda}' \\ ab'b\bar{b}\bar{b}'}} \int \frac{d^3p_{\bar{c}} d^3p_{c'} d^3k}{(2\pi)^9 \sqrt{2\omega}} (2\pi)^3 \delta(p_{c'} + k - p_{\bar{c}}) d_{p_{c'}\lambda'b'}^+ d_{p_{\bar{c}}\bar{\lambda}\bar{b}} \frac{\lambda_{b'\bar{b}}^a}{2} a_{k\lambda a}^+ \\ \delta_{b,b'} \delta_{\lambda,\lambda'} \chi_{\lambda'}^+ \left[\frac{\vec{\varepsilon}^{(\mu_g)} (\vec{p}_{\bar{c}} + \vec{p}_{c'})}{2m} + \frac{\vec{\varepsilon}^{(\mu_g)} \vec{k}}{m_g} + \frac{i\vec{\sigma}}{2m} (\vec{\varepsilon}^{(\mu_g)} \wedge \vec{k}) \right] \chi_{\bar{\lambda}} dp_c \delta_3(p_c - p_{c'}) . \quad (43)$$

The (3S) charmonium meson is given in the non-relativistic approximation by :

$$|\psi(3S)\rangle = \sum_{\lambda\lambda'm_{c\bar{c}}a} \int \frac{d^3p_c d^3p_{\bar{c}}}{(2\pi)^6} \frac{1}{\sqrt{3}} \chi_{\lambda\lambda'}^{\lambda_{c\bar{c}}} (2\pi)^3 \delta_3(p_c + p_{\bar{c}}) \\ \psi_{c\bar{c}} \left(\frac{p_c - p_{\bar{c}}}{2} \right) \langle \ell_{c\bar{c}}, m_{c\bar{c}} S_{c\bar{c}} \lambda_{c\bar{c}} | J_{c\bar{c}} M_{c\bar{c}} \rangle b_{p_c \lambda a}^+ d_{p_{\bar{c}} \lambda' a}^+ \quad (44)$$

where :

$$\psi_{c\bar{c}}(p_c) = \left(\frac{16\pi^3 R_{c\bar{c}}^3}{\Gamma(3/2)} \right)^{1/2} \frac{4}{\sqrt{30}} L_2^{1/2}(R_{c\bar{c}}^2 p_c^2) y_{\ell_{c\bar{c}}}^{m_{c\bar{c}}}(\vec{p}_c) \exp \left[-\frac{1}{2} (R_{c\bar{c}}^2 p_c^2) \right] . \quad (45)$$

$L_2^{1/2}(x)$ being the Laguerre polynomial of the second order, since the 3S charmonium is a second radial excitation wave function.

To compute the charmonium-hybrid transition we will calculate the matrix element:

$$I^{\ell_{q\bar{q}}(\ell_g)} \equiv \langle c\bar{c}g(\text{hybrid}) | H_{\text{trans}} | \psi(3S) \rangle . \quad (46)$$

We obtain :

$$I^{\ell_{c'\bar{c}'}(\ell_g)} = 2Bg \sum_{\substack{\lambda,\lambda',m_g,m_{c\bar{c}},m_{c'\bar{c}'} \\ \mu_g,\lambda,\bar{\lambda},\lambda'}} \int \frac{d^3p_c d^3p_{c'}}{(2\pi)^6 \sqrt{2\omega}} \frac{2}{\sqrt{3}} \\ \chi_{\lambda'}^+ \left[\vec{\varepsilon}^{(\mu_g)} \frac{(\vec{p}_c + \vec{p}_{c'})}{2m} + \vec{\varepsilon}^{(\mu_g)} \frac{(\vec{p}_c - \vec{p}_{c'})}{m_g} + \frac{i\vec{\sigma}}{2m} (\vec{\varepsilon}^{(\mu_g)} \wedge (\vec{p}_c - \vec{p}_{c'})) \right] \chi_{\lambda} \\ \psi_{\ell_{c'\bar{c}'}}^{m_{c'\bar{c}'}} \left(\frac{\vec{p}_c + \vec{p}_{c'}}{2} \right) \psi_{\ell_g}^{m_g}(\vec{p}_c - \vec{p}_{c'}) \psi_{\ell_{c\bar{c}}}^{m_{c\bar{c}}}(\vec{p}_c) \langle \ell_{c\bar{c}} m_{c\bar{c}} S_{c\bar{c}} \lambda_{c\bar{c}} | J_{c\bar{c}} M_{c\bar{c}} \rangle \\ \langle \ell_g m_g 1 \mu_g | J_g M_g \rangle \langle \ell_{c'\bar{c}'} m_{c'\bar{c}'} J_g M_g | L m \rangle \langle L m S_{c'\bar{c}'} \lambda_{c'\bar{c}'} | J M \rangle \\ \langle \frac{1}{2} \lambda \frac{1}{2} \bar{\lambda} | S_{c\bar{c}} \lambda_{c\bar{c}} \rangle \langle \frac{1}{2} \lambda' \frac{1}{2} \bar{\lambda}' | S_{c'\bar{c}'} \lambda_{c'\bar{c}'} \rangle . \quad (47)$$

Where :

$$B = \frac{(16\pi^3)^{3/2} R_{c'\bar{c}'}^{5/2} R_g^{3/2} R_{c\bar{c}}^{3/2}}{\pi \sqrt{30} (\Gamma(3/2))^2 \Gamma(5/2)} \quad (48)$$

We have used in the calculation generating function $w(x, t)$ of the Laguerre polynomials, which is easier to handle than the Laguerre polynomials themselves :

$$w(x, t) = (1 - t)^{-a-1} e^{-xt/(1-t)} = \sum_{x=0}^{\infty} L_x^\alpha(x) t^x \quad , \quad |t| < 1 \quad . \quad (49)$$

From (49) and (47) we obtain :

$$I^{\ell_{c'\bar{c}'}=1} = 2\alpha_s \left[\left(\frac{1}{2m} T^1 + \frac{1}{m_g} T^2 \right) 3\delta_{S_{c\bar{c}}, S_{c'\bar{c}'}} \delta_{L,0} \delta_{J, J_{c\bar{c}}} \delta_{M, M_{c\bar{c}}} \right. \\ \left. + \frac{i}{2m} T^2 (-2i) \delta_{J, J_{c\bar{c}}} \delta_{M, M_{c\bar{c}}} \delta_{L, S_\sigma} \right] \quad (50)$$

and

$$I^{\ell_g=1} = 2\alpha_s \frac{i}{2m} T^4 (-2i) \delta_{J, J_{c\bar{c}}} \delta_{M, M_{c\bar{c}}} \delta_{L, S_\sigma} \quad (51)$$

where :

$$\begin{aligned} T^1 &= -0.0009446 \text{ GeV}^2 \quad , \\ T^2 &= -0.0005347 \text{ GeV}^2 \quad , \\ T^3 &= -0.0003679 \text{ GeV}^2 \quad , \\ T^4 &= -0.0007595 \text{ GeV}^2 \quad . \end{aligned} \quad (52)$$

Finally the transition amplitudes between the 3S charmonium and the hybrid meson are listed in table 5 :

P	C	ℓ_g	$\ell_{c'\bar{c}'}$	J_g	$S_{c'\bar{c}'}$	L	$\langle c\bar{c}g H_{tr} c\bar{c} \rangle (\alpha_s \text{Mev})$
-	-	0	1	1	1	0	-5.6
-	-	0	1	1	1	1	-0.629
-	-	0	1	1	1	2	0
-	-	1	0	1	0	1	-0.89

Table 5: Predicted transition amplitudes between the 3S charmonium and the hybrid mesons in MeV times α_s .

These matrix elements turn out to be very small, of the order of 1 MeV. Using these transition matrix elements and the fact that $E_+ = 4160 \text{ Mev}$ and $E_- = 4040 \text{ Mev}$ are the expected eigenvalues of the transition Hamiltonian (41), it is easy to compute the mixing angle between the (3S) charmonium and the $c\bar{c}g$ hybrid meson:

$$\tan(\theta) = \frac{2|H_{12}|}{\sqrt{(E_+ - E_-)^2 - 4|H_{12}|^2}} \quad (53)$$

where H_{12} is one of the transition matrix elements given in table 5. The result are listed in table 6 :

l_g	J_g	$l_{c\bar{c}}$	$S_{c\bar{c}}$	L	$H_{12}(MeV)$	$\tan(\theta)$	θ
0	1	1	1	0	5.6	0.0937	0.0934
0	1	1	1	1	0.629	0.0105	0.0105
0	1	1	1	2	0	0	0
1	1	0	0	1	0.89	0.0148	0.0148

Table 6: Predicted mixing angles between the (3S) charmonium and the $c\bar{c}g$ hybrid meson

These mixing angles turn out to be very small, because the transition matrix elements are much smaller than the level spacing between $\psi(4040)$ and $\psi(4160)$. This feature invalidates the the (OCP) scenario which needs a rather large mixing angle as exemplified in eq. (40). A small mixing angle leaves us with two states very similar to the starting ones: one hybrid, weakly coupled with the photon and thus incompatible with any of the two $\psi(4040)$ and $\psi(4160)$ states, and only one $\psi(3S)$ whcih cannot account for the latter two states.

The smallness of the transition matrix elements is due to the quasi-orthogonality between the conventional $\psi(3S)$ and the wave functions of the $c\bar{c}$ pair in the hybrid, namely (1S) or (1P) state. The recoil of the gluon and the difference of the radii is not enough to eliminate this quasi-orthogonality suppression. As a check we have also computed the matrix elements between the (1S) charmonium and the hybrids, and we obtain mixing amplitudes which are two orders of magnitude larger than with the (3S).

5 Conclusion

We have considered the $J^{PC} = 1^{--}$ hybrid $c\bar{c}g$ mesons in a quark and gluon constituent model. Using for simplicity a chromoharmonic potential (12), we have computed the spectrum and wave functions of these hybrids, their decay widths into ground state mesons and finally their mixing with $\psi(3S)$.

Restricting ourselves to the lightest states ($l_g + l_{c\bar{c}}=1$) we have found one hybrid meson with an orbital excitation between the gluon and the $c\bar{c}$ system ($l_g = 1$), and three with an orbital excitation between the c and the \bar{c} ($l_{c\bar{c}} = 1$). The former is stable for decay into ground state meson (the only opened channels around 4.1 GeV). The latter have so large widths that they do not really exist as resonances. What happens here is simply that nothing prevents the constituent gluon to decay instantaneously into a quark pair. Within the chromoharmonic model, these two classes do not mix, leaving alive only one hybrid meson ($l_g = 1$) which does not decay. This patter of two types of hybrids, one forbidden, teh other allowed to decay into ground state mesons is quite similar to the

results for the flux tube model, except for a reverse ordering of masses. We do not know what decay width the latter model would predict for the allowed decay.

We have thus considered the latter as a candidate for the OCP scenario which explains anomalies in the pair $\psi(4040) - \psi(4160)$ as the result of a large-angle mixing between a hybrid meson and the $\psi(3S)$ charmonium. However, the very small mixing Hamiltonian matrix elements (due to the quasi-orthogonality of the $(3S)$ $c\bar{c}$ wave function in the charmonium and the $(1S)$ or $(1P)$ in the hybrid) and the very small resulting angles lead us to exclude the OCP scenario in view of the observed mass splitting $\psi(4040) - \psi(4160)$.

Finally it should be noted that a more realistic potential might induce a large mixing between the two classes of hybrids, resulting in very large widths for all hybrids and finally no real resonant hybrid. Only a small attraction in the two meson continuum due to a very short-lived hybrid “state” might then remain. This might look like what has been seen in the analogous light meson 1^{-+} sector where a possibly hybrid induced “activity” may have been seen, as noted in [3].

If such a large mixing between the different classes of hybrids does not exist, the search for $l_g = 1$ hybrids remains an important task, although made difficult by the absence of coupling to the ground state mesons.

Anyhow a new explanation for the $\psi(4040) - \psi(4160)$ mystery has still to be found in view of the difficulties of the OCP scenario.

Acknowledgements

It is a pleasure to thank Suh-Urk Chung for valuable discussions. We are particularly indebted to Philip Page for very useful comments and for a careful and critical reading of the draft.

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