Model reference adaptive control with fractional derivative

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Abstract- Over the few last years the idea of introducing fractional calculus and systems in adaptive control has found a great interest, for the benefit one can win in the performances given by such systems. The main idea is to impose a fractional structure that can improve the behavior of the closed loop. By the mean of introducing a fractional order derivation at the plant output, and an adapted fractional model reference, we can achieve this aim, because of the best dynamical stability and robustness against perturbations.

Key words: fractional derivative, fractional order system, Model Reference Adaptive Control.

1. Introduction

There has been considerable research in the area of fractional order systems and their application in control (Sun, 1990) (Oustaloup, 1983-1991) (Loiseau,1998), and since few years many researches have focused on the introduction of fractional order operators in adaptive control, and especially on MRAC (Model Reference Adaptive Control)(Vinagre, 2002). In this approach adaptive algorithms allow the control of systems on which few information are known.

First, the use of fractional model reference in the adaptive scheme has shown an improvement in system dynamics, due to the best model reference dynamical properties(Ladaci, 2002). Then the introduction of fractional integration has proven the ability of fractional algorithms to guarantee stability with a highest level of performance then the integer order algorithms (it depends on the choice of the Integration fractional order).

The originality of this contribution is the use of both of the too approaches discussed bellow, by introducing a fractional model reference and a fractional derivation (which from an algebraic point of view, can be assimilated to a advanced fractional integration in the adaptation loop).

The use of the derivative action has always been done with a lot of care in automatics, because, in the presence of noise it can damage the desired performance or worst the system stability. However in the fractional derivative case, which is a long memory process (Hotzel,1998), we show that the perturbation rejection is achieved at a satisfactory level, joining several other research conclusions on the robustness of fractional systems versus perturbation.

This paper is structured as follows:

Section 2 introduces the fractional order systems, with both integration and derivation definitions. Section 3 then introduces the model reference adaptive control (MRAC) problem and the use of fractional operators in the adaptation algorithm. An example is presented in section 4, with comments on simulation results. Finally some concluding remarks are presented in section 5.

2. Fractional order systems

The analysis in Bode plot of many natural processes, like transmission lines, dielectric polarisation impedance, interfaces, cardiac rhythm, spectral density of physical wave, some types of noise (Van Der Ziel, 1950) (Duta, 1981) , has allowed to observe a fractional slope. This type of process is known as *1/f process* or fractional order system. The used description equation into frequency domain of these processes is given as follows :

$$X(s) = \frac{k}{(1+\frac{s}{pt})^m} \qquad (1)$$

with m : fractional exponent.
pt : fractional pole which is the cut frequency.
s : Laplace operator.

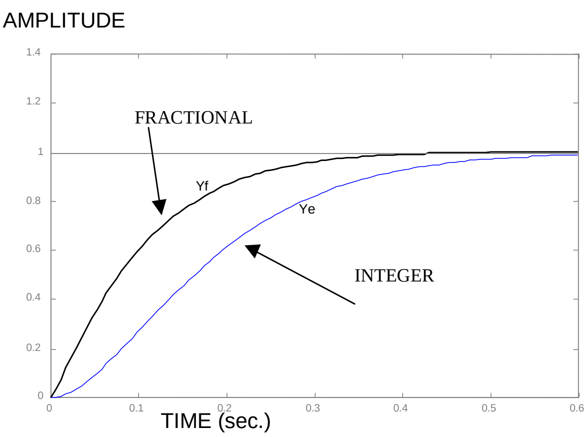

Figure 1. Comparative step response (fractional / integer)

2.1. Fractional integration and derivative

Fractional derivation and integration are classical tools in engineering, they have been used in mechanics since at least the 1930's and in electrochemistry since the 1960's. In control field, interesting works have been

For complex systems where puissance is varying from a real number to another, they are represented by a multiple poles function with fractional power :

$$X(s) = \frac{1}{\prod_{i=1}^{n}(1+s/pt_i)^{m_i}} \qquad 0 \le m_i \le 1 \qquad (2)$$

where $(1/pt_i)$: relaxation times.

Performances:

Many precedent works have shown that fractional systems present best qualities, in response time and in transition dynamic stability (Sun, 1990). In fact, for a second order system, represented by the following transfer function :

$$G(s) = \frac{1}{\left(\frac{s^2}{w_n^2} + 2\xi\frac{s}{w_n} + 1\right)^m} \qquad (3)$$

with : w_n=10 rd/s, $\xi = 0.95$

The step responses for the integer case (m=1) and the fractional order one (m=0.55) are given in figure1, and show the gain in fastness.

achieved in the soviet union (Brin, 1962), and an increasing interest began mainly after the contributions done in France (Oustaloup, 1995).

2.1.1 Definition of fractional integration

(Fliess, 1997-1998)

Let $\alpha \in C$, $\Re(\alpha) > 0$, $c \in R$ and f a locally integrable function defined on [c,+∞[. The α order integral of f , of lower bound c is defined as :

$$I_c^\alpha f(t) \overset{\Delta}{=} \int_c^t \frac{(t-\tau)^{\alpha-1}}{\Gamma(\alpha)} f(\tau)d\tau \qquad (4)$$

with $t \ge c$, and Γ is the Euler function. The formula (4) is called *Riemann-Liouville Integral*.

Usually, the control loop is discreet, and we use a sampled approximation of (4) given by :

$$I_c^\alpha f(k\Delta) = \frac{\Delta}{\Gamma(\alpha)} \sum_{\tau=0}^{k-1} (k\Delta - \tau\Delta)^{\alpha-1} f(\tau\Delta) \qquad (5)$$

with, Δ : Sampling Period.

In practice we have found it necessary to add a constant real **c**, to obtain a best dynamical behaviour, so :

$$I_c^{\alpha} f(k\Delta) = \underline{c} + \frac{\Delta}{\Gamma(\alpha)} \sum_{\tau=0}^{k-1} (k\Delta - \tau\Delta)^{\alpha-1} f(\tau\Delta) \quad (6)$$

From a theoretical point of view, we know that fractional systems are infinite dimensional systems (Hotzel, 1998). The $I_c^{\alpha} f(k\Delta)$ depends on all the past values of f(.) since the initial instant. With the sampling process the is a certain loss of information due to the sampling period and the impossibility to compute $I_c^{\alpha} f(k\Delta)$ until the last period k in (6) because we will have a divide by zero.

2.1.2. Definition of Fractional Order derivative:

It is defined as follows, by consideration of the equality: $D^{\alpha} f(t) = I_c^{-\alpha} f(t)$

We can write:

$$D^{\alpha} f(t) = \frac{d^{\alpha}}{dt^{\alpha}} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f(t - kh) \quad (7)$$

And assuming that: $D^{\alpha} f(t) \approx D_h^{\alpha} f(t)$, we have:

$$D_h^{\alpha} f(t) = h^{-\alpha} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(kh - jh) \quad (8)$$

With:

$$\binom{\alpha}{j} = \frac{\alpha!}{j!(\alpha - j)!} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha - j + 1)}$$

Where

$\binom{\alpha}{0} = 1$ and Γ : Euler function.

Computation of coefficients:

The z-Transform of fractional derivation can be obtained as follows:

$$(1-z)^{\alpha} = \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} z^k = \sum_{k=0}^{\infty} \omega_k^{(\alpha)} z^k \quad (9)$$

with $\omega_0^{(\alpha)} = 1$;

$$\omega_k^{(\alpha)} = \left(1 - \frac{\alpha+1}{k}\right) \omega_{k-1}^{(\alpha)} \qquad k = 1,2,3...$$

2.2 Linear approximation of fractional order Transfer function:

For the purpose of our approach we need to use an integer order model approximation of the fractional order model reference in order to implement the adaptation algorithm, for this aim we have used the so-called singularity function method (Charef, 1991) , and precisely for the case interesting our approach that is a fractional second order system of the form (3) with m a positive real number such that 0<m<0.5 We can approximate

$$H(s) = \frac{1}{\left(\frac{s^2}{\omega^2} + 2\xi \frac{s}{\omega} + 1\right)^m}$$

by the function:

$$H(s) = \frac{\left(\frac{s}{\omega} + 1\right)\left(\frac{s}{\omega + 1}\right)^{\beta}}{\left(\frac{s^2}{\omega^2} + 2\alpha \frac{s}{\omega} + 1\right)} \quad (10)$$

With $\alpha = \xi^m$ And $\beta = 1 - 2m$, which also can be represented by the function,

$$H(s) = \frac{\left(\frac{s}{\omega} + 1\right)}{\left(\frac{s^2}{\omega^2} + 2\alpha \frac{s}{\omega} + 1\right)} \frac{\prod_{i=1}^{N-1} (1 + \frac{s}{z_i})}{\prod_{i=1}^{N} (1 + \frac{s}{p_i})} \quad (11)$$

The singularities are given by:

$p_j = (ab)^{j-1} a.z_1 \qquad j = 1, 2, 3,... N$

$z_i = (ab)^{i-1} z_1 \qquad i = 2, 3,... N-1$

with, $z_1 = w\sqrt{b}$, $a = 10^{\frac{\varepsilon_p}{10(1-\beta)}}$, $b = 10^{\frac{\varepsilon_p}{10\beta}}$,

$\beta = \frac{\log(a)}{\log(ab)}$

and ε_p : Tolerated error in dB

The order of approximation N is computed by fixing the frequency band of work, specified by $\omega_{\max}$, so that:

$p_{N-1} < \omega_{\max} < p_N$ Which leads to:

$$N = \text{integer part of}\left[\frac{\log\left(\frac{\omega_{max}}{p_1}\right)}{\log(ab)}+1\right]+1 \qquad (12)$$

H(s) can be then be written under a parametric shape function of order N+2:

$$H(s) = \frac{b_{m0}s^N + b_{m1}s^{N-1} + ... + b_{mN}}{s^{N+2} + a_{m1}s^{N+1} + ... + a_{mN+2}} \qquad (13)$$

a_{mi} and b_{mi} are calculated from the singularities p_i, z_i and α, ω.

3. Applying in adaptation algorithm

3.1. Model Reference Adaptive Control

(Astrom, 1995), (Landau, 1979)

It is one of the more used approaches of adaptive control, in which the desired performance is specified by the choice of a reference model. Adjustment of parameters is achieved by the mean of the error between the output of the plant and the model reference output. Which can be represented like in figure 2.

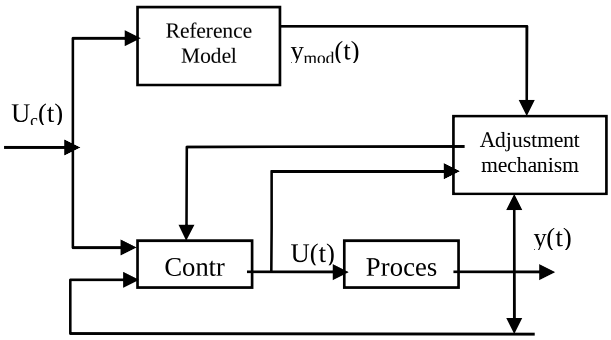

Figure 2. Direct Model Reference Adaptive Control

3.2. M.I.T. Rule:

We consider a closed loop system where the controller has an adjustable parameter vector θ. A model which output is y_m specifies the desired closed loop response. Let **e** be the error between the closed loop system output **y** and the model one $\mathbf{y_m}$, one possibility is to adjust the parameters such that the cost function:

$$J(\theta) = \frac{1}{2}e^2 \qquad (14)$$

be minimised. In order to make **J** small it is reasonable to change parameters in the direction of negative gradient **J**, so:

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J}{\partial\theta} = -\gamma e\frac{\partial e}{\partial\theta} \qquad (15)$$

3.3. Introducing Fractional Operators:

Let the Reference model G_{mod} be a fractional order transfer function of second order like in (3). We introduce also a fractional derivation bloc at the process output, as it is represented in figure 3. We assume that the relative degree of the siso process transfer function n is known. It means that $G(s) = \frac{Num(s)}{den(s)}$ with

Deg(Den(s))-Deg(Num(s))=n (16)

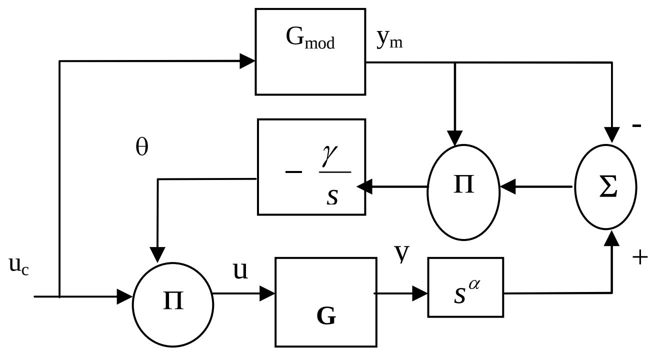

Figure 3. Bloc diagram of Adaptation algorithm

So when $|s|$ is of high value, we can write:

$$G_{mod}(s) = \frac{y_m}{u_c} = \frac{1}{\left(\frac{s^2}{w_n^2} + 2\xi\frac{s}{w_n} + 1\right)^m} \approx \frac{1}{s^{2m}}$$

and also, $G(s) = \frac{y}{u} \approx \frac{1}{s^n}$ (17)

So in order to compare $y_m(t)$ with $\frac{d^\alpha y(t)}{dt^\alpha}$ we must assure this equality,

$\frac{s^\alpha}{s^n} \approx \frac{1}{s^{2m}}$ in other words:

$\alpha = n - 2m$

(18)

4. Example

Taking: $G = \dfrac{10}{s+5}$,

With a model transfer function

$$G_{\text{mod}} = \frac{1}{\left(s^2 + 1.8s + 1\right)^m}$$

Where $\omega = 1$; $\xi = 0.9$ This transfer function is sampled and approximated to an integer order model by the singularity function method (13).

For m = 0.4, 0.35, we obtain the simulation results given in figures 4. And 5.

Comments

- The stability of the closed loop is maintained, with a good level of performances.
- When m<<0.5 which means that $\alpha \approx n$ (for example m= $0.1 \Rightarrow \alpha = 0.8 \approx 1$) the rejection of perturbation is not achieved and the response is very bad, as the order of derivation is quite integer.
- However for higher values of m there is an effective rejection of perturbation as expected because as we can see in (7), the calculus of the fractional derivative is dependent of all the history of the signal, which will moderate the effect of last variations.

5. Conclusion

A fractional Model adaptive control algorithm which includes the use of fractional derivation was presented that can guarantee the closed loop stability with a good level of performances and high ability to reject perturbation. The simulations showed the improvement of performances of the adaptive control algorithm even when there are perturbations. This approach is interesting also in the case of fractional order processes. The stability and robustness conditions of such systems are under study.

6. References

Astrom, K.J., & Wittenmark, B. (1995) Adaptive Control, Addison-Wesley, USA.

Brin, I. A. (1962) On the stability of certain systems with distributed and lumped parameters. Automat. Remote Control, vol. 23, 798-807.

Charef A., (1991) Analysis and Synthesis of fractal systems, Ph.D. Thesis, Drexel University.

Duta P. & P.M. Horn, (1981) Low frequency fluctuations in solids : 1/f noise. Review of modern physics, Vol. 53, No 3.

Fliess M. & Hotzel R. (1998) On linear systems with a fractional derivation: Introductory theory with examples" in Mathematics and computers in Simulation 45, 385-395

Fliess M. & Hotzel, R. (1997) Sur les systèmes linéaires à dérivation non entière. C.R.Acad. Sci. Paris, série II b, (signal, informatique), vol. 324, 99-105.

Hotzel R., (1998) "Contribution à la théorie structurelle de la commande des systèmes linéaires fractionnaires", Thèse de Doctorat, Université de Paris-Sud, Centre d'Orsay.

Ladaci S. & Charef A. (2002) Commande Adaptative à modèle de référence d'ordre Fractionnaire d'un bras artificiel. Revue Sciences et Communication, ENSET Oran Algeria, N°1, October, 53-55.

Landau, Y.D., (1979) Adaptive Control : The model reference Approach, Marcel Dekker, New York.

Loiseau, J.J., & Mounier, H. (1998) Stabilisation de l'équation de la chaleur commandée en flux, ESAIM, Proc., 131-144.

Oustaloup A., (1983) Systèmes asservis d'ordre fractionnaire. Masson, Paris.

Oustaloup A., (1991) La commande CRONE. Hermès, Paris.

Oustaloup A., (1995) La dérivation non entière, Hermès, Paris.

Sun, H., & Charef, A. (1990) Fractal System-A time domain Approach , Annals of Biomedical Ing.Vol :18, 597-621.

Van Der Ziel, A. (1950) On the noise spectra of semiconductor noise and of flikker effects. Physica. 16, 359-372.

Vinagre, B. M., & Petras, I., & Podlubny, I., & Chen, Y.Q. (2002) Using Fractional Order Adjustment Rules and Fractional Order Reference Models in Model-Reference Adaptive Control. In Nonlinear Dynamics 29, 269-279.

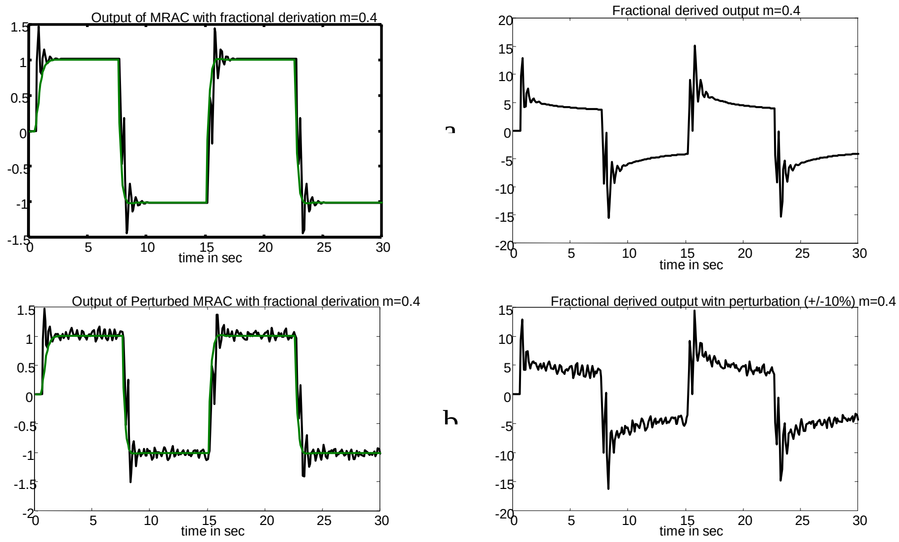

Figure 4. Output of the process and its fractional derivation for m = 0.4
a-without perturbations
b-with perturbation of 10% (Amplitude).

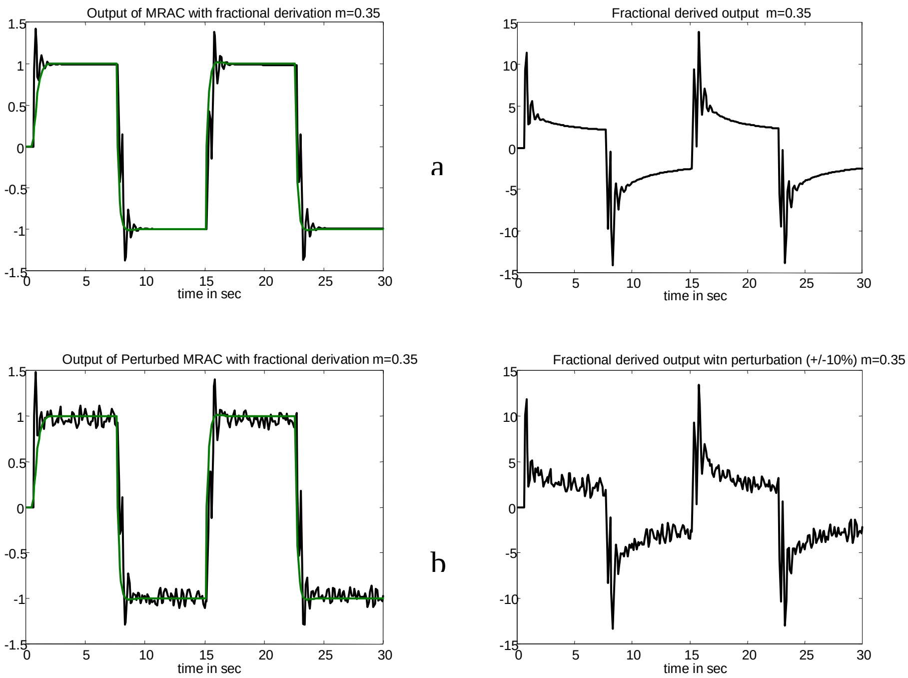

Figure 5. Output of the process and its fractional derivation for m = 0.35
a-without perturbations
b-with perturbation of 10% reference signal amplitude.