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Aperture Integration and Wavelet-based Moment Method Analysis of Horn Antenna For Millimeter-Wave Applications

M. LASHAB[*], C. ZEBIRI[**] and F. BENABDELAZIZ[***]

[*]*Département d'électronique, Université de Skikda, Algérie.*
Lashabmoh@yahoo.fr

[**]*Département d'électronique, Université de Sétif, Algérie.*
Zebiri@netcourrier.com

[***]*Département d'électronique, Université de Mentouri, Constantine, Algérie*
Benabdelaziz2003@yahoo.fr

Abstract: Horn Antenna for millimeter-wave application has been analyzed by two methods, the first one is an asymptotic one which is the aperture integration based on the geometrical optics (GO), the second one is a numerical method based on the well known moment method and improved by wavelets. The aim of this work is first to introduce the application of wavelet in electromagnetic scattering, secondly a comparison of the two method of analysis is presented to show the limit of use of the asymptotic method an to emphasize on the fact that moment method is an exact method which is improved by the introduction of wavelet by reduction of both memory space and processing time, this is illustrated in this work.
Key words: Moment Method, Wavelets, Horn antenna, Aperture integration.

INTRODUCTION

Design of feeds for large reflector antennas such as horns became crucial and more computing time and memory space especial for millimeter-wave applications. The analysis of radiation pattern can be achieved by asymptotic methods such as the Aperture Integration which is based on Geometrical and Physical Optics Technique [1,2], some work have been done using numerical solutions for electromagnetic scattering problems such as Moment Method [3], this is one of the most powerful numerical techniques for solving electromagnetic problems, however this method has always suffered from memory and computation time, due to digitalized integral equations resulting for very dense impedance matrix.

Still author numerical method have been applied to improve the accuracy such as the adaptive integral method [4,5,6]. Here in this work we trying to compare results obtained form the aperture integration method which is an asymptotic method, with the wavelet technique which is based on the moment method for future combination between these two methods, because for very large size structures, even very powerful methods may face the problem of memory space and computing time. Many work have

been curried out using wavelet to solve electromagnetic problems the results are very promising [7,8,9,10]. The authors have also presented and published works on the use of wavelets for solving electromagnetic problems [11]. The pyramidal horn antenna is excited at the mode TE_{10} and the frequency of work is 10 GHz.

The theoretical contents is expressed in the next section, which consists of three subsections, The first subsection is dedicated to the Aperture integration and problem description, the second subsection is about integral equation and Moment Method formulation, the last subsection is for orthogonal wavelet expansion.

1. Formulation

1.1 Aperture Integration

The pyramidal horn understudy is presented in figure 1, since the aperture integration is based on the geometrical optics, we assume that the incident filed is formed over the aperture plane, and the aperture field has a uniform distribution. The radiated field considered here is in TE_{10} mode with y directed electric field. The electric far field radiated by the aperture may be written as [12].

$$E_\theta = jk \frac{1+\cos\theta}{2}\left[f_x\cos\phi + f_y\sin\phi\right]\frac{e^{-jkr}}{2\pi r} \quad (1)$$

And,

$$E_\phi = jk \frac{1+\cos\theta}{2}\left[f_y\cos\phi - f_x\sin\phi\right]\frac{e^{-jkr}}{2\pi r} \quad (2)$$

Where (r,θ,ϕ) are the spherical coordinates points in the radiation zone, k is the free space propagation constant, $f = f_x\hat{x} + f_y\hat{y}$ is the Fourier transformer of the field at the aperture $E_x(x,y,o)$, which may be written as:

$$f(\theta,\phi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}E_x(x,y,0)e^{jk_x x}e^{jk_y y}dxdy \quad (3)$$

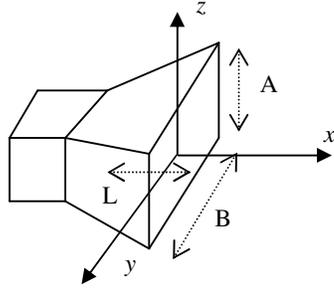Where, $k_x = k\sin\theta\cos\phi$, $k_y = k\sin\theta\sin\phi$.

Figure 1. Pyramidal Horn antenna in 3D.

Since the field outside the aperture is assumed to be zero equation (3) can be written as

$$f(\theta,\phi) = \int_{-A/2}^{+A/2}\int_{-B/2}^{+B/2}E_x(x,y,0)e^{jk_x x}e^{jk_y y}dxdy \quad (4)$$

Where A, B are the dimensions of the broad aperture of the pyramidal horn. The electric field at the aperture is currying only TE$_{10}$ mode, it can be found as [12].

$$E_x(x,y,o) = E_{10}\cos(\frac{\pi y}{B})e^{j\beta_{10}L} \quad (5)$$

Where E$_{10}$ is the field magnitude, β_{10} is the phase constant for the mode TE$_{10}$. The radiation pattern for the aperture antenna is given with spectral dependence as [1]

$$\overline{E}(r,\theta,\phi) = \frac{je^{-jkr}}{2\lambda r}(1+\cos\theta)(\cos\theta\hat{\theta}$$
$$- \sin\phi\hat{\phi}).f(\theta,\phi) \quad (6)$$

The radiation pattern is obtained when replacing equation (5) into (4) and (4) into (6). The radiation

field in the E-plane (x-z plane), E_θ at fixed radial position can be obtained by replacing $\phi = 0$ at equation (6) in this case $E_\phi = 0$. The radiation field in the H-plane (y-z plane) , E_ϕ at fixed radial position is obtained by replacing $\theta = 0$ at equation (6) in this case $E_\theta = 0$.

1.2 Moment Method Formulation
1.2.1   Integral equation

The Pyramidal Horn is studied in 2D as shown in figure 2, the surface of this latter is considered to be perfect conductor. Using the boundary condition, the scattered field may be written as an integral of the induced current and the 2D Green's function as [12].

$$\frac{k\eta}{4}\int_C J_y(\rho)H_0^2\left(k|\rho-\rho'|\right)dc' = E_x^i(\rho) \quad (7)$$

Where C is the contour and $H_0^{(2)}(k|\rho-\rho'|)$ is Hankel function of the second kind zero order. In explicit manner the variable radius is:

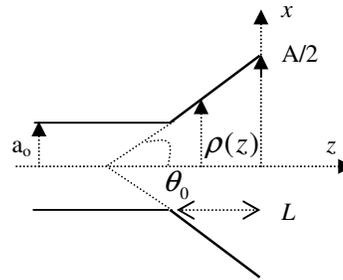$$\rho(z) = a_o + \frac{((A/2)-a_o)(z+L)}{L} \quad (8)$$

Figure 2. Pyramidal Horn in 2D

Equation (7) can be expressed as:

$$\frac{k\eta}{4}\int_{\theta=-\theta_0}^{\theta=\theta_0} J_y(\rho)H_0^{(2)}(kR)|\rho(\theta)|d\theta = E_x^i(\rho) \quad (9)$$

Where, $R = |\rho-\rho'|$. The excitation here is defined by equation (5).

1.2.2 Moment method

The induced current can be found when using the Moment Method [12], and expand this current in a series of N basis functions, given as an inner product [11].

$$\sum_{n=1}^{N}\left\langle g_m, \frac{\omega\mu_0}{4} \int_{\theta=-\theta_0}^{\theta=\theta} F_n . H_0^{(2)}(R) . |r'(\theta)| . d\theta \right\rangle I_n =$$

$$\left\langle g_m, E_x^{inc}(r) \right\rangle, \text{ for } m=1....M. \qquad (10)$$

Where the Fn are the basis functions and the In are the unknown constants, And using N similar testing functions (g_m). After then the matrix equation is solved and the unknowns are determined.

1.3 Wavelet expansions

The basis and testing functions are presented as a superposition of wavelets at several scales including the scaling function. A Galerkin method is then applied, where the set of basis functions used to present the current function, are used as weighting functions. The wavelets used here are Haar basis an orthogonal type. Its study is useful from theoretical point of view, because it offers an intuitive understanding of many multi-resolution properties. Furthermore, due to its simplicity Haar wavelets are widely employed. The scaling function is $\phi(x)$, and the mother wavelet function is $\psi(x)$, these are defined as [13]:

$$\phi(x) = \begin{cases} 1, & \text{for } 0 \le x \le 1 \\ 0, & \text{Otherwise} \end{cases} \qquad (11)$$

And

$$\psi(x) = \begin{cases} 1, & \text{for } 0 \le x \le 1/2 \\ -1, & \text{for } 1/2 \le x \le 1 \\ 0, & \text{Otherwise} \end{cases} \qquad (12)$$

Also the scaling and the mother wavelets functions are defined by:

$$\phi_{jn}(x)=2^{j/2}\phi(2^j x - n) \qquad (13)$$

$$\psi_{mn}(x)=2^{m/2}\psi(2^m x - n) \qquad (14)$$

Where ($m \text{ or } j$) are the resolution level and (n) is the translation factor. The vector space V^j linear span of ϕ_{jn} for $j = 0,1…$ And $n = 0,1….2^j -1$. The vector space W^j linear span of ψ_{jn} for $j = 0,1….$ And $n = 0,1….2^j -1$. The property of $W^j \subseteq V^{j+1}$ holds the relation between the vector spaces of different functions by :

$$V^{j+1} = V^j \oplus W^j \qquad (15)$$

The above equation states that the sub-space W^j is the orthogonal complement of V^j in a larger subspace V^{j+1}, which means for a given function $f \in R^N$ with N samples or N Dimensional vector, the projection of this function into the orthogonal basis is as fellow :

$$V^k = V^0 \oplus W^0 \oplus W^1 ..... \oplus W^{k-1}, \qquad (16)$$

For k = 0,1…N-1

And can be expressed in a inner product as :

$$f = \left\langle f, v_0 \right\rangle v_0 + \left\langle f, v_1 \right\rangle v_1 + ... + \left\langle f, v_{N-1} \right\rangle v_{N-1} \qquad (17)$$

The wavelets are applied directly upon the integral equation. The density of current will be represented as a linear combination of the set wavelets functions and scaling functions as fellow:

$$Jz(x)=\sum_n a_n . \phi_{j,n}(x) + \sum_{m=j}^{2^j-1}\sum_n c_{m,n}\psi_{m,n}(x) \qquad (18)$$

For example for N=2, equation (18) can be expressed as:

$$Jx(x) = a_0\phi_{0,0} + c_{0,0}\psi_{0,0} + c_{1,0}\psi_{1,0} + c_{1,1}\psi_{1,1} +$$

$$c_{2,0}\psi_{2,0} + c_{2,1}\psi_{2,1} + c_{2,2}\psi_{2,2} + c_{2,3}\psi_{2,3} \qquad (19)$$

The number of wavelets used here are 8 and the matrix corresponding is of $8x8 = 64$ elements. The main mathematical properties which enable sparse matrix generation are the orthogonally and the vanishing moment. A function $\psi(x)$ is said to have vanishing moment of N order if:

$$\int_{-\infty}^{+\infty} x^n . \psi(x) . dx = 0 \qquad \forall n = 0,1....(N-1) \qquad (20)$$

The fact that the wavelets are orthogonal and the presence of vanishing moment, this is enabling sparse matrix production. When applying equation (18) into (10) we obtain the set of matrix equation as fellow:

$$\begin{bmatrix} [Z_{\phi,\phi}] & [Z_{\phi,\psi}] \\ [Z_{\psi,\phi}] & [Z_{\psi,\psi}] \end{bmatrix} \begin{bmatrix} a_n \\ c_{m,n} \end{bmatrix} = \begin{bmatrix} \left\langle E_z^{inc}, \phi_{j,n} \right\rangle \\ \left\langle E_Z^{inc}, \psi_{m,n} \right\rangle \end{bmatrix} \qquad (21)$$

Where:

$$[Z_{\phi,\phi}] = \left\langle \phi_{j,n}, \frac{\omega\mu_0}{4} \int_{\theta=-\theta_0}^{\theta=\theta_0} \phi_{j,n} . H_0^{(2)}(R) . |r'(\theta)| . d\theta \right\rangle \qquad (22)$$

$$\left[Z_{\phi,\psi} \right] = \left\langle \phi_{j,n}, \frac{\omega\mu_0}{4} \int\limits_{\theta=-\theta_0}^{\theta=\theta_0} \psi_{m,n} . H_0^{(2)}(R) \left| r'(\theta) \right| d\theta \right\rangle \quad (23)$$

$$\left[Z_{\psi,\phi} \right] = \left\langle \psi_{m,n}, \frac{\omega\mu_0}{4} \int\limits_{\theta=-\theta_0}^{\theta=\theta_0} \phi_{j,n} . H_0^{(2)}(R) \left| r'(\theta) \right| d\theta \right\rangle \quad (24)$$

$$\left[Z_{\psi,\psi} \right] = \left\langle \psi_{m,n}, \frac{\omega\mu_0}{4} \int\limits_{\theta=-\theta_0}^{\theta=\theta_0} \psi_{m,n} . H_0^{(2)}(R) \left| r'(\theta) \right| d\theta \right\rangle \quad (25)$$

Since Galerkin Method employs the same testing functions and basis function, in the same manner is the equation for incident field is expressed. After then the unknown constants are determined, and the current density can be found using equation (21).

2. Numerical Results

A computer program has been coded in Matlab language for the technique described above, the wavelet employed is constructed from Haar orthogonal wavelet with vanishing moment N=6, the lowest resolution level is chosen $2^j = 2^6 = 128$, since 128 wavelets are involved, a system of matrix (of 128 x128 elements) is generated. Figure 3 show the radiation pattern for E-plane, we can notice almost good agreement between the two methods, especially the first lobe.
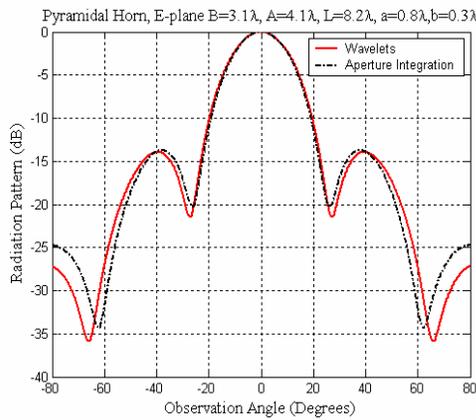

Figure 3. Radiation Pattern for pyramidal Horn, E-plane, A= 1.4λ, B=2.2λ, L=3.14 λ, F=12 Ghz

The result presented in figure 4, show very interesting comparison, for the H-plane the two methods aperture integration and the wavelet-based moment method the radiation pattern is almost the same, far away from the edges the radiation pattern is well predicted by the aperture integration.

The sparsification of the impedance matrix with respect to the chosen threshold and the wavelet number is presented in table 1.

One can notice from the table 1, that increasing wavelet number can improve sparsification and lower

threshold decrease sparsification. Although accuracy of the results are obtained for very low threshold a compromise is necessary.

Threshold	10^{-2}			
Wavelet Number	32	64	128	512
Sparsity(%)	79.50	85.11	89.34	91.03
Threshold	10^{-3}			
Wavelet Number	32	64	128	512
Sparsity(%)	36.14	75.33	78.12	80.56
Threshold	10^{-4}			
Wavelet Number	32	64	128	512
Sparsity(%)	58.22	68.02	71.23	73.05

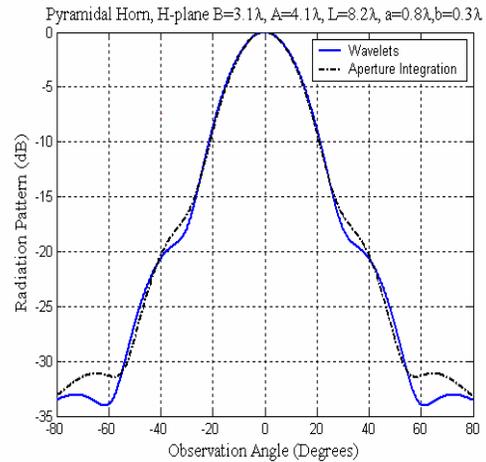**Table 1.** Sparsification with respect to the threshold and the wavelet number.

Figure 4. Radiation pattern for pyramidal Horn, H-plane, A= 1.4λ, B=2.2λ, L= 3.14λ, F=12 Ghz

In this paper the use of Haar wavelet leads to a matrix sparsity of 76,2%, for a threshold of 0.1%, which means the moment matrices were rendered sparse by thresholding to zero all matrix elements whose magnitude was less then 0.1% of all the maximum magnitude of all matrix entries.

3. Conclusion

The analysis of radiation pattern of large horn has been presented using two methods which are the aperture integration and wavelet-based moment method. The results obtained are very closing especial for the main lobe and the second or third one. Aperture integration can be used in combination with wavelet-based moment method if the edges are avoided. The results obtained when using wavelets can be improved when using large number of wavelets and low threshold the computing time be greater, a

compromise have to be done.

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