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Synthesis of a Unknown Inputs Proportional Integral Observer for TS Fuzzy Models

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Abstract— **In this paper, synthesis conditions of a proportional integral observer for a Takagi-Sugeno (TS) fuzzy models subject to unknown inputs and unmeasurable decision variables are established. These unknown inputs affect both state and output of the system. The synthesis of this observer is based on hypothesis that the unknown inputs are under the polynomials form with bounded norm of their k^{th} derivatives. The Lyapunov theory and L_2–gain technique are used to develop the stability conditions of such observers in LMIs formulation. A numerical example is proposed to validate the proposed design conditions.**

Index Terms— **Unmeasurable decision variables, Unknown inputs reconstruction, Proportional Integral Observer, TS fuzzy models.**

I. Introduction

MOST physical systems with a nonlinear dynamic behavior are approximated by Takagi-Sugeno (TS) fuzzy models [1]. These last are described by fuzzy IF-THEN rules which represent the local linear input-output relations of a nonlinear system [2]. The structure of TS fuzzy models allows representing the local dynamics of each fuzzy implication by a linear model. Thus, the global behavior of the nonlinear system is obtained by using the nonlinear activation functions defining the contribution of each linear model. There are two types of decision variables which intervene in the activation functions, measurable, the case of inputs or outputs variables, or unmeasurable the case of unavailable state variables. It was shown in [3] that if the state variable is used as the decision variable, the fuzzy models obtained represent a large class of nonlinear systems. For this raison, it is better to consider the fuzzy models with unmeasurable decision variables. Thus, it is interesting to exploit this approach in observer's synthesis of state estimation for diagnostic and control of reel systems.

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The study of stability and stabilization is closely related to the state estimation of T-S fuzzy models [4]–[6]. Therefore, the convergence of the state estimation errors are developed by the Lyapunov theory and formulated under the linear matrix inequality (LMI) constraints [7]. In the case of unmeasurable decision variables we can mention the works developed in [8], [9],which deal the design of sliding mode observers and robust observer for unknown input respectively, and in [10] the design of output feedback controllers. In [11], based on the L_2–gain technique, the design conditions of observers in LMI form are given.

The majority of physical systems are subject to disturbances which can have as source: measurement noises, modeling uncertainties, sensors and actuators faults. These disturbances, considered as unknown inputs, have adverse effects on the normal behavior of the real system and their estimates can be used to conceive systems of diagnostic and control. Then robust observers are proposed to simultaneously estimate states and actuator faults for different class of nonlinear system [12]–[17].

We differentiate two types of observers for the unknown inputs estimation: by decoupling (UIO) and proportional integral (PIO). The first type allows to completely decoupling the unknown inputs so that the estimation error becomes insensitive to these inputs. We can name for example, the works of [13] for measurable decision variables and [9], [14] for unmeasurable decision variables. Nevertheless, the reconstruction of these unknown inputs is drowned in the noise amplified by the derivation of output signal which is generally soiled by measurement noise. In the other hand, the second type, proportional integral observer, allows estimating not only the system states but also the unknown inputs by an integral action. We can name the works developed in [15], [16] for the uncertain TS fuzzy models to unmeasurable decision variables with constant unknown inputs. Also, the unknown inputs estimation in a polynomial form is studied in [17] with measurable decision variables and in [12] for a class of nonlinear descriptor system. On the other hand, the works in [18], [19] deal the unmeasurable decision variables as a disturbed system and as Lipschitz constraint, respectively.

In comparison with previous studies, we compensate the influence of the unmeasurable decision variables by adding a parameter in the synthesis conditions of the proportional integral observer. We note that, this PIO called also observer to sliding mode estimates simultaneous the states and the unknown inputs. Moreover, the introduction of an integral gain as another parameter helps to reduce the conservatism results because it constitutes an additional degree of freedom to be determined.

In this work a modified PIO is considered for TS fuzzy models subject to unknown inputs. Indeed, supplementary parameters are introduced to compensate the effect due to unmeasurable decision variables and unknown polynomials inputs. The proposed approach deals the k^{th} derivatives of unknown inputs as bounded norm. The design conditions are established on the basis of the Lyapunov stability theory, the L_2–gain technique and LMIs formulation.

This work is organized as follows. In section II, we present the structure of TS fuzzy models with unmeasurable decision variables. These models are subject to unknown inputs which can affect the dynamics and the output signals. In section III, the structure and synthesis of the proposed PIO are presented. Finally in section IV, a numerical example is given to show the good estimation of both state and unknown inputs.

II. STRUCTURE OF THE UNKNOWN INPUTS TS FUZZY MODELS

Consider a TS Fuzzy model with unmeasurable decision variables and subject to unknown inputs on the dynamics and on the output:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x)\big(A_i x(t) + B_i u(t) + E_i v_x(t)\big) \\ y(t) = Cx(t) + Ev_y(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^{n_u}$ is the known input vector, $v_x(t) \in R^{n_{v_x}}$ and $v_y(t) \in R^{n_{v_y}}$ are the unknown inputs, and $y(t) \in R^{n_y}$ represents the output vector. $A_i \in R^{n\times n}$ are the state matrices, $B_i \in R^{n\times n_u}$ are the input matrices, $E_i \in R^{n\times n_{v_x}}$ and $E \in R^{n_y\times n_{v_y}}$ are the unknown inputs matrices, and $C \in R^{n_y\times n}$ is the output matrix. The $\mu_i(x)$ represent the activation functions which depend on the state $x(t)$ of the system. These functions have the following properties:

$$\begin{cases} \sum_{i=1}^{r} \mu_i(x) = 1, & \forall\, t \geq 0 \\ 0 \leq \mu_i(x) \leq 1, & \forall\, i \in \{1, .. r\} \end{cases}$$

where r represents the local models number.

The TS fuzzy models (1) can be rewritten in the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x)\big(A_i x(t) + B_i u(t) + F_i v(t)\big) \\ y(t) = Cx(t) + Fv(t) \end{cases} \tag{2}$$

where $v(t) = \begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix}$, $F_i = [E_i \quad 0]$ and $F = [0 \quad E]$.

Hypothesis 1: The unknown input $v(t)$ is a polynomial in time function, of *k-1* degree, which its k^{th} derivative is not equal to zero and is bounded norm, noted v_0. With the following notations:

$$\begin{cases} \dot{v}(t) = v_1(t) \\ \dot{v}_1(t) = v_2(t) \\ \vdots \\ \dot{v}_{k-1}(t) = v_k(t) \\ \|v_k(t)\| \leq v_0 \end{cases} \tag{3}$$

In the diagnosis framework, the assumption of the unknown inputs $v(t)$ in polynomials form allow considering a large range of faults. These last can be assimilated as independent actuator and sensor faults affecting a physical system.

III. STRUCTURE OF THE UNKNOWN INPUTS PIO

The considered PIO is as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x})\Big(A_i\hat{x}(t) + B_i u(t) + F_i\hat{v}(t) + K_{Pi}\big(y(t) - \hat{y}(t)\big)\Big) + z_x(t) \\ \hat{y}(t) = C\hat{x}(t) + F\hat{v}(t) \\ \dot{\hat{v}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x})K_{Ii}\big(y(t) - \hat{y}(t)\big) + \hat{v}_1(t) + z_v(t) \\ \dot{\hat{v}}_j(t) = \sum_{i=1}^{r} \mu_i(\hat{x})K_{Ii}^{j}\big(y(t) - \hat{y}(t)\big) + \hat{v}_{j+1}(t) + z_{vj}(t) \quad \text{for} \quad j: 1 \ldots k-1 \end{cases} \tag{4}$$

where $K_{Pi} \in R^{n\times n_y}$ and $K_{Ii} \in R^{n_v\times n_y}$, $K_{Ii}^{j} \in R^{n_v\times n_y}$ represent the proportional and integral gains, respectively. The variables $z_x(t)$ and $z_v(t)$, $z_{vj}(t)$ are introduced in order to compensate the influence of the unmeasurable decision variables.

The proposed PIO (4) not only allows the states estimation of TS fuzzy models (2) subjected to unknown inputs but also the reconstruction of these inputs in presence of unmeasurable decision variables.

Based on the *hypothesis 1*, the TS fuzzy models (2) and the PIO (4) can be written respectively under the following augmented forms:

$$\begin{cases} \dot{x}_a(t) = \sum_{i=1}^{r} \mu_i(x)(\bar{A}_i x_a(t) + \bar{B}_i u(t)) + Vv_k(t) \\ y(t) = \bar{C}x_a(t) \end{cases} \tag{5}$$

$$\begin{cases} \dot{\hat{x}}_a(t) = \sum_{i=1}^{r} \mu_i(\hat{x})\Big(\bar{A}_i\hat{x}_a(t) + \bar{B}_i u(t) + \bar{K}_i\big(y(t) - \hat{y}(t)\big)\Big) + V\hat{v}_k(t) + z(t) \\ \hat{y}(t) = \bar{C}\hat{x}_a(t) \end{cases} \tag{6}$$

$$\text{where } x_a(t) = \begin{bmatrix} x(t) \\ v(t) \\ v_1(t) \\ \cdots \\ v_{k-1}(t) \end{bmatrix},\ \hat{x}_a(t) = \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \\ \hat{v}_1(t) \\ \cdots \\ \hat{v}_{k-1}(t) \end{bmatrix},\ z(t) = \begin{bmatrix} z_x(t) \\ z_v(t) \\ z_{v1}(t) \\ \cdots \\ z_{vk-1}(t) \end{bmatrix} \tag{7a}$$

with

$$e_a(t) = x_a(t) - \hat{x}_a(t),\ e_{ay} = y(t) - \hat{y}(t) \tag{7b}$$

$$\bar{A}_i = \begin{bmatrix} A_i & F_i & 0 & 0 & \dots & 0 \\ 0 & 0 & I_{n_v} & 0 & \dots & 0 \\ 0 & 0 & 0 & I_{n_v} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & I_{n_v} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix},\ \bar{K}_i = \begin{bmatrix} K_{Pi} \\ K_{Ii} \\ K_{Ii}^1 \\ \dots \\ K_{Ii}^{k-1} \end{bmatrix},\ V = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ I_{n_v} \end{bmatrix} \tag{7c}$$

$$\bar{C} = [C \quad F \quad 0 \quad \dots \quad 0] \tag{7d}$$

and I_{n_v} is the identity matrix of dimension $n_v = n_{v_x} + n_{v_y}$.

The dynamics of state estimation error $e_a(t)$ is represented by:

$$\dot{e}_a(t) = \sum_{i=1}^{r} \mu_i(\hat{x}) \mathcal{A}_i e_a(t) + \bar{\Delta} A x_a(t) + \bar{\Delta} B u(t) + V \Delta v_k(t) - z(t) \tag{8}$$

where

$\mathcal{A}_i = \bar{A}_i - \bar{K}_i \bar{C}$, $\bar{\Delta} A = \sum_{i=1}^{r} \bar{\mu}_i \bar{A}_i$, $\bar{\Delta} B = \sum_{i=1}^{r} \bar{\mu}_i \bar{B}_i$, $\bar{\mu}_i = \mu_i(x) - \mu_i(\hat{x})$, $\Delta v_k = v_k - \hat{v}_k$.

Remark 1: The convex sum property of activation functions allows to write $-1 < \bar{\mu}_i < 1$, hence the variables matrices $\bar{\Delta} A$ and $\bar{\Delta} B$ are bounded and the following properties are verified:

$$\|\bar{\Delta} A\| \le \delta_1,\quad \delta_1 = \sum_{i=1}^{r} \delta_{1i} \quad \text{and} \quad \|\bar{\Delta} B\| \le \delta_2,\quad \delta_2 = \sum_{i=1}^{r} \delta_{2i} \tag{9}$$

with $\delta_{1i} > 0$ and $\delta_{2i} > 0$ are respectively the Euclidian norms of $\bar{A}_i$ and $\bar{B}_i$.

Remark 2: Since the k^{th} derivative of the unknown input is bounded, its estimated is also bounded and therefore the difference $\Delta v_k(t)$ is bounded. The transfer of the $\Delta v_k(t)$ towards increased estimation error $e_a(t)$ is minimized by L_2–gain technique [7].

Lemma 1: For any X and Y of appropriate dimensions matrices, the following property is verified:

$$X^T Y + Y^T X \le \lambda X^T X + \lambda^{-1} Y^T Y \quad with \quad \lambda > 0$$

Theorem: The system (8) *is asymptotically stable if there exist a matrix* $P = P^T > 0$, *matrices* $\bar{N}_i$ *and the positive scalars* α *and* α_0 *such as:*

$$\text{Min}\ \bar{\gamma}$$

$$\begin{bmatrix} P\bar{A}_i + \bar{A}_i^T P - \bar{N}_i \bar{C} - \bar{C}^T \bar{N}_i^T + \alpha_0 \delta_1^2 I + I & PV & P \\ V^T P & -\bar{\gamma} I & 0 \\ P & 0 & -\alpha I \end{bmatrix} < 0 \tag{10}$$

where $\gamma = \sqrt{\bar{\gamma}}$ *and parameters* $\bar{A}_i$, $\bar{C}$ *and* δ_1 *are defined in* (7c), (7d) *and* (9).

The observer parameters in (4) *are given by:*

$$\bar{K}_i = P^{-1} \bar{N}_i$$

and

$$\begin{cases} z = 0 & if \quad |e_{a_y}| < \varepsilon \\ z = \sigma_1 \delta_1^2 \dfrac{\hat{x}_a^T \hat{x}_a}{2 e_{a_y}^T e_{a_y}} P^{-1} \bar{C}^T e_{a_y} + \sigma_2 \delta_2^2 \dfrac{u^T u}{2 e_{a_y}^T e_{a_y}} P^{-1} \bar{C}^T e_{a_y} & if \quad |e_{a_y}| \ge \varepsilon \end{cases}$$

with variables $\hat{x}_a$, z, e_{ay} *and the parameter* δ_2 *are defined in* (7a), (7b), (9) *respectively and* $\sigma_1 = \left(\frac{\alpha_0}{\lambda}\right)$, $\sigma_2 = \left(\frac{\alpha \alpha_0}{\alpha(1+\lambda) - \alpha_0}\right)$ *where* λ *is a small scalar arbitrarily fixed and* ε *is a very small positive threshold.*

Consider the Lyapunov function $V(t) = e_a^T(t) P e_a(t)$ where $P = P^T > 0$. The obtained LMI conditions are only sufficient and can be restrictive when the number of local models increases.

Remark 3: The conservatism results introduced by quadratic functions can be reduced by new Lyapunov candidate functions appropriate for TS fuzzy models, in particular the nonquadratic Lyapunov functions. Indeed, it is well known that these new functions give areas of solutions more vast than those quadratic functions.

Proof: The time-derivative of $V(t) = e_a^T(t) P e_a(t)$ allows writing:

$$\begin{aligned} \dot{V} = \sum_{i=1}^{r} \mu_i(\hat{x}) (e_a^T (\mathcal{A}_i^T P + P \mathcal{A}_i) e_a) + x_a^T \bar{\Delta} A^T P e_a + e_a^T P \bar{\Delta} A x_a \\ + u^T \bar{\Delta} B^T P e_a + e_a^T P \bar{\Delta} B u + \Delta v_k^T V^T P e_a + e_a^T P V \Delta v_k \\ - z^T P e_a - e_a^T P z \end{aligned} \tag{11}$$

Then the *lemma 1* allows us to write:

$$\begin{aligned} \dot{V} \le \sum_{i=1}^{r} \mu_i(\hat{x}) (e_a^T (\bar{\mathcal{A}}_i^T P + P \bar{\mathcal{A}}_i) e_a) + \lambda_1 \delta_1^2 x_a^T x_a + \lambda_1^{-1} e_a^T P^2 e_a \\ + u^T \bar{\Delta} B^T P e_a + e_a^T P \bar{\Delta} B u + \Delta v_k^T V^T P e_a + e_a^T P V \Delta v_k \\ - z^T P e_a - e_a^T P z \end{aligned} \tag{12}$$

Taking account (7b) and using again the *lemma 1,* we obtain:

$$\begin{aligned} \dot{V} \le \sum_{i=1}^{r} \mu_i(\hat{x}) (e_a^T (\mathcal{A}_i^T P + P \mathcal{A}_i + \alpha_0 \delta_1^2 I + \alpha^{-1} P^2) e_a) + \sigma_1 \delta_1^2 \hat{x}_a^T \hat{x}_a \\ + \sigma_2 \delta_2^2 u^T u + \Delta v_k^T V^T P e_a + e_a^T P V \Delta v_k \\ - 2 e_a^T P z \end{aligned} \tag{13}$$

with $\alpha_0 = \lambda_1(1+\lambda)$, $\alpha^{-1} = (\lambda_1^{-1} + \lambda_2^{-1})$, $\sigma_1 = \lambda_1(1+\lambda^{-1}) = \left(\frac{\alpha_0}{\lambda}\right)$, $\sigma_2 = \lambda_2 = \left(\frac{\alpha \alpha_0}{\alpha(1+\lambda) - \alpha_0}\right)$ $(\alpha \ne \lambda_1)$.

Substituting the variable expression of z into (13), we get:

$$\begin{aligned} 2 e_a^T P z &= 2\, e_a^T P\, \sigma_1 \delta_1^2 \frac{\hat{x}_a^T \hat{x}_a}{2 e_{a_y}^T e_{a_y}} P^{-1} \bar{C}^T e_{a_y} \\ &\quad + 2\, e_a^T P \sigma_2 \delta_2^2 \frac{u^T u}{2\, e_{a_y}^T e_{a_y}} P^{-1} \bar{C}^T e_{a_y} \\ &= \sigma_1 \delta_1^2 \hat{x}_a^T \hat{x}_a + \sigma_2 \delta_2^2 u^T u \end{aligned}$$

with $e_{a_y} = \bar{C} e_a$ and $e_{a_y}^T = e_a^T \bar{C}^T$. Hence, the expression (13) becomes:

$$\dot{V} \le \sum_{i=1}^{r} \mu_i(\hat{x}) e_a^T (\bar{\mathcal{A}}_i^T P + P\bar{\mathcal{A}}_i + \alpha_0 \delta_1^2 I + \propto^{-1} P^2) e_a + \Delta v_k^T V^T P e_a + e_a^T P V \Delta v_k \quad (14)$$

The system (8) is stable and verifies the L_2–gain condition ($\|e_a(t)\|_2 < \gamma \|\Delta v_k(t)\|_2$, $\gamma > 0$) if the following condition is satisfied:

$$\dot{V} + e_a^T e_a - \gamma^2 \Delta v_k^T \Delta v_k < 0 \quad (15)$$

Then condition (15) leads to:

$$\sum_{i=1}^{r} \mu_i(\hat{x}) e_a^T (\Psi i + \propto^{-1} P^2) e_a + \Delta v_k^T V^T P e_a + e_a^T P V \Delta v_k - \gamma^2 \Delta v_k^T \Delta v_k < 0 \quad (16)$$

with $\Psi_i = \bar{\mathcal{A}}_i^T P + P\bar{\mathcal{A}}_i + \alpha_0 \delta_1^2 I + I$ $(i = 1, \dots r)$. Then, the condition (16) is verified if the following matrix inequalities hold:

$$\begin{bmatrix} \Psi i + \propto^{-1} P^2 & PV \\ V^T P & -\gamma^2 I \end{bmatrix} < 0 \quad (17)$$

Apply the Schur complement to (17) with the variable change $\bar{\gamma} = \gamma^2$ and $\bar{N}_i = P\bar{K}_i$, we get (10). The proof is completed. □

The resolution of these constraints allows to obtain the unknown input PIO gains $\bar{K}_i = P^{-1}\bar{N}_i$ and as well as the scalars α and α_0 with $\gamma = \sqrt{\bar{\gamma}}$.

In the following, a numerical example is given in order to validate the proposed approach.

IV. Numerical Example

Consider a nonlinear system represented by a TS Fuzzy models of two local models (r =2) subjected to unknown inputs and unmeasurable decision variables:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(x)\big(A_i x(t) + B_i u(t) + E_i v_x(t)\big) \\ y(t) = Cx(t) + E v_y(t) \end{cases}$$

The matrices numerical values are:

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -6 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix},\ E_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -3 & 2 & 2 \\ 5 & -8 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.5 \\ 1 \\ 0.25 \end{bmatrix},\ E_2 = \begin{bmatrix} 1 \\ 0.5 \\ 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix},\ E = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}.$$

The activation functions are defined by:

$$\begin{cases} \mu_1(x) = \frac{1 - \tanh(x_3)}{2} \\ \mu_2(x) = 1 - \mu_1(x_3) \end{cases}$$

The resolution of LMIs constraints (10) leads to the PIO gains:

$$\bar{K}_i = \begin{bmatrix} K_{Pi}^T K_{Ii}{}^T & K_{Ii}^{1\,T} & K_{Ii}^{2\,T} & K_{Ii}^{3\,T} \end{bmatrix}^T$$

$$Z = \begin{bmatrix} Z_x{}^T & Z_v{}^T Z_{v1}{}^T & Z_{v2}{}^T & Z_{v3}{}^T \end{bmatrix}^T$$

The gains of the PIO are given in the table 1:

TABLE 1

λ=6	$\alpha = 2.9421\ 10^6$	$\alpha_0 = 9.5935\ 10^{-4}$
i	1	2
K_{pi}	$\begin{bmatrix} 35.325 & 54.937 \\ 28.440 & 54.766 \\ 40.660 & 60.906 \end{bmatrix}$	$\begin{bmatrix} 033.632 & 134.064 \\ 016.118 & 144.702 \\ 033.599 & 182.889 \end{bmatrix}$
K_{Ii}	$\begin{bmatrix} 00856.275 & 001342.780 \\ 00020.716 & -000045.138 \end{bmatrix}$	$\begin{bmatrix} 0725.836 & 003619.274 \\ 0027.964 & -000072.328 \end{bmatrix}$
K_{Ii}^1	$\begin{bmatrix} 10211.775 & 015894.330 \\ 00421.191 & -000940.800 \end{bmatrix}$	$\begin{bmatrix} 08662.403 & 043107.092 \\ 00519.249 & -001513.531 \end{bmatrix}$
K_{Ii}^2	$\begin{bmatrix} 70005.056 & 108347.547 \\ 04441.490 & -009437.562 \end{bmatrix}$	$\begin{bmatrix} 59412.955 & 294825.422 \\ 05495.438 & -015109.276 \end{bmatrix}$
K_{Ii}^3	$\begin{bmatrix} 237267.77 & 365496.464 \\ 024421.61 & -045796.208 \end{bmatrix}$	$\begin{bmatrix} 201467.87 & 996657.032 \\ 029883.76 & -070595.598 \end{bmatrix}$

The considered unknown inputs, with *4th* derivatives bounded norm, are given in figure 1. These results are obtained with an attenuation gain $\gamma = 0.4$, $\varepsilon = 10^{-3}$ and $v_0 = 88.623$. The simulation results are carried out with the initial conditions: $x_0 = [-1 \quad 0.5 \quad 1]$ and $\hat{x}_0 = [1 \quad -0.5 \quad -1]$ where Fig. 1 represents the unknown inputs and their estimates and in Fig. 2 their dynamic errors. Fig.3 and Fig. 4 represent the dynamic errors of the states and the outputs, respectively. The simulation results show the good estimation of both state and unknown inputs.

V. Conclusion

In this work, a proportional integral observer for TS fuzzy model subjected to unknown inputs and unmeasurable decision variables is proposed. The considered unknown inputs, which influence both state and output signals, are assumed in polynomial form and their *kth* derivatives are bounded norm. Based on Lyapunov theory, the design conditions are established in LMIs form. In order to validate this approach a numerical example is given. The proposed example shows the efficiency of the derived conditions since both state and unknown inputs are well estimated. This approach constitutes a good technique for the development of faults diagnosis algorithm.

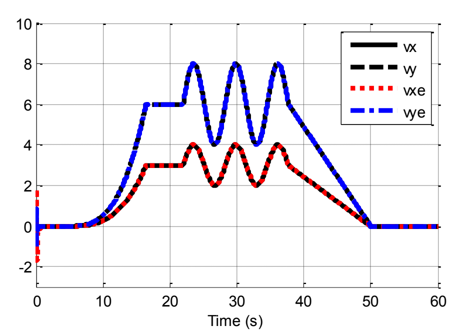

Figure 1. The unknown inputs and their estimates

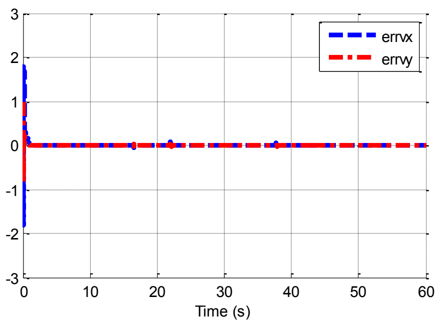

Figure 2. The errors between unknown inputs and their estimates

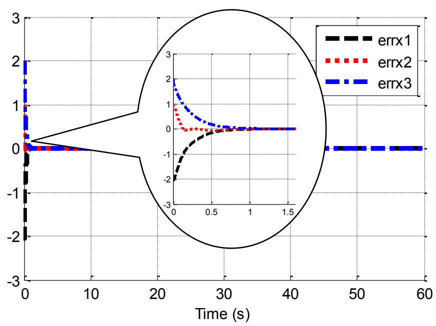

Figure 3. The errors between states and their estimates

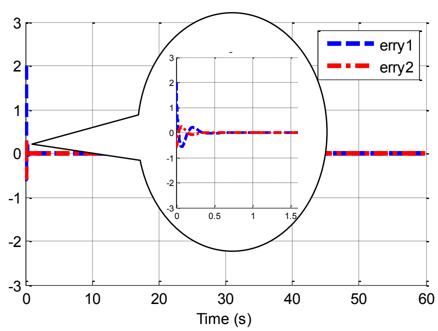

Figure 4. The errors between outputs and their estimates

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