Microchannel Fluid Flow and Heat Transfer By Lattice Boltzmann Method

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ABSTRACT

Micro flow has become a popular field of interest due to the advent of micro electromechanical systems (MEMS). In this work, the lattice Boltzmann method, a particle-based approach, is applied to simulate the two-dimensional micro channel fluid flow.

We simulated fluid flow and heat transfer inside microchannel, the prototype application of this study is micro-heat exchangers. The main incentive to look at fluidic behaviour at micron scale is that micro devices tend to behave much differently from the objects we are used to handling in daily life. The choice of using LBM for micro flow simulation is a good one owing to the fact that it is based on the Boltzmann equation which is valid for the whole range of the Knudsen number. Slip velocity and temperature jump boundary conditions are used for the microchannel simulations with Knudsen number values covering the slip flow. The lattice Bhatnagar-Gross-Krook single relaxation time approximation was used. The results found are compared with the Navier-Stokes analytical and numerical results available in the literature and good matches are observed.

Keywords: Lattice Boltzmann method, micro channel, MEMS, heat transfer

1.INTRODUCTION

With the rapid development of Micro-electro-mechanical systems (MEMS), flow and heat transfer in micro devices have become an area that receives a significant attention, due to their applications in various of industrial fields and many other scientific applications.

These MEMS also contains many micro fluidics devices such as micro channels, micro pumps, micro nozzles micro-heat exchangers, micro-turbines, and micro filters. Flows through micro channels are the most common configuration in all of the biomedical applications [4]. Both experimental and computational efforts have been undertaken to understand the specific features of the micro scale flows. The mechanism of the microscopic flow and heat transfer is quite different from that of macroscopic counterparts because the characteristic length of the flow is comparable order of magnitude to the mean free path, and the intermolecular interactions are manifested. The microscopic flows are usually characterized by a dimensionless parameter, the Knudsen number Kn which describes different regimes of fluid flow depending on it. For Kn>10, the flow system can be considered to be a free molecular flow. A flow is considered a continuum for Kn< 0,001. The intermediate values of Kn, for 0,1<Kn<10 are associated with a transition flow regime, while those within the range of 0,001<Kn<0,1 represent a slip flow regime. As Kn increases this rarefaction effect becomes truly significant. Another main effect is compressibility, basically gas flows are compressible effect. Viscous heating and thermal creep [5], [15] are other important effects. Another important effect in micro flow is the slip boundary conditions. This has been proven by a number of experimental studies that revealed the presence of slippage[10]. Leading to the breakdown of the no-slip assumption [11].

Due to the advent of (MEMS) and the micro fluidic devices inside the systems, the study of micro flow has become an important field of interest for many researchers. A large number of methods have emerged in the past few decades, which include molecular dynamics (MD), direct simulation Monte Carlo (DSMC), dissipative particle dynamics (DPD), smooth-particle hydrodynamics (SPH), Lattice-gas cellular automata (LGCA) and Lattice Boltzmann method (LBM). Each of these simulation methods has its limitations and can only be used within specific length and time scales [10].The molecular dynamics (MD) is computationally intensive and suitable for certain scale ranges only, this approach is presently only applicable to small systems[8]. A particularly promising method is the (LBM), which is a mesoscale method connecting the microscopic and macroscopic descriptions of the flow dynamics [5]. (LBM) is relatively recent technique that has been shown to be as accurate as traditional (CFD) methods having the ability to integrate arbitrarily complex geometries at a reduced computational cost. (LBM) is relatively new and promising method for the solution of nonlinear partial differential equations and simulation of fluid flows.

Historically the (LBM) evolves from (LGCA) which belongs to the class of cellular automata that are used for simulation of the fluid flow phenomena [3]. It represents an idealization of the physical system in which space and time both are discrete. In 1986, Frisch, Hasslacher, and Pomeau and Wolfram proposed the first two-dimensional lattice gas automaton model for the specific purpose of computational fluid dynamics [12]. In 1988, a proposal to use the lattice Boltzmann equation to simulate fluid flow problems was made for the first time. (LBM) has been used as a computational fluid dynamics (CFD) tool for well over two decades. Unlike the lattice gas automata (LGA) and molecular dynamics (MD) approaches, the LB method simulates a flow system by tracking the evolution of particle distributions, not single particles. It differs from traditional (CFD) in that (LBM) does not directly solve for the macroscopic variables found in the Navier–Stokes equations. The LB method, due to its kinetic nature, has a linear convection operator, and the local nature of the variables make it well suited for parallel computers [16].

Whereas Boltzmann equation (BE) is valid for all Kn flows, it is difficult to solve except for the free-molecule regime, mostly because of the nonlinear collision term [18]. It should be noted that in many micro and nano applications the (LB) method cannot be used, thus prompting to the use of more expensive (MD),or multi scale methods [7],[8].

Many researchers have carried out simulations of micro flows using (LB) method. Nie et al [4], [9] used the (LBM) with bounce back boundary condition to simulate 2D micro channel and micro lid driven cavity flow. Both the specular bounce back rule and the extrapolation scheme were employed to generate the slip effect by Lim et al [2]. Tang et al [4] applied kinetic theory based boundary conditions to study gaseous slip flow in micro scale geometries. Zhang et al [17] used tangential momentum accommodation coefficient (TMAC) to describe the gas-surface interactions. It was concluded that the (LBM) is an efficient tool for simulation of micro flow. All the above models discuss mainly isothermal micro flows. Recently, thermal lattice Boltzmann method has attracted much attention because of its potential applications as well as practical importance in engineering designs and energy related problems, such as solar collectors, thermal insulation, cooling of electronic components, heat exchangers, air heating systems for solar dryers. For this purpose there have been several authors who studied (TLBM). Shu et al [6] focused on the implementation of diffusion reflection boundary conditions for the TLBM and analyze the kinetic boundary condition and get similar boundary conditions as that of Karniadakis and Beskok [5]. Tian et al [6] used the Maxwell first-order slip boundary conditions. Surface structure, wettability and nanosacle roughness are some of the factors that have been recognized to affect slippage phenomena [10]. N.Asproulis and D.Drikakis used (MD) simulation to study the slip lengh's dependency on the wall stiffness[10]. Another investigation was carried out by N.Asproulis and D.Drikakis, they studied the combined effects of surface stiffness and wall particle's mass on the slip length [11], aiming to enhance understanding of the momentum and energy transfer across solid-liquid interfaces.

This work deals with the analysis of the (TLBM) for simulating microchannel laminar forced convection within the slip flow regime without viscous dissipation, which is encountered in microchannel flows. Double distribution functions are used to simulate the velocity and the temperature fields. In order to capture the velocity slip and the temperature jump of the wall boundary, we adopted the Maxwell first order slip boundary condition based on macro variables without thermal creep [19]. A slip regime with $kn < 0.1$ where (N.S) equation is still valid is considered.

2.THERMAL LB MODEL FOR MICRO-FLOW AND NUMERICAL SIMULATION

The (TLBM) can be derived from the discrete Boltzmann equation with Bhatnagar- Gross Krook (BGK) collision operator.
Following proposed thermal lattice Boltzmann (BGK) model [6], [13], [19] which employ two discrete evolution equations for flow and temperature distribution function, f_i and g_i, respectively, we simulated thermal part using another distribution function for the temperature. In this paper a square grid and D_2Q_9 model is used for both flow and temperature. The governing equations for flow and temperature can be written as:

$$f_i(X + c_i\Delta t, t+\Delta t) - f_i(X,t) = -\frac{1}{\tau_f}\left[f_i(X,t) - f_i^{eq}(X,t)\right] \quad (1)$$

$$g_i(X + c_i\Delta t, t+\Delta t) - g_i(X,t) = -\frac{1}{\tau_g}\left[g_i(X,t) - g_i^{eq}(X,t)\right] \quad (2)$$

The Δx and Δt are the lattice grid spacing and the time step; τ_f and τ_g are the momentum and internal energy relaxation, time respectively.
The discrete velocities c_i for the two dimensional nine bit square lattice model are given by:

$$c_i = (0,0) \quad (i=0)$$

$$c_i = \left[\cos\frac{(i-1)\pi}{2}, \sin\frac{(i-1)\pi}{2}\right]c \quad (i=1\text{-}4) \quad (3)$$

$$c_i = \left[\cos\frac{(i-1)\pi}{2}, \sin\frac{(i-1)\pi}{2}\right]c \quad (i=5\text{-}8)$$

f_i and g_i are the density distribution function and internal energy density distribution function, respectively, f_i^{eq} and g_i^{eq} are their corresponding equilibrium distribution functions which are given by:

$$f_i^{eq} = \omega_i\rho\left[1 + \frac{3(c_i.u)}{c^2} + \frac{9(c_i.u)^2}{2c^4} - \frac{3(u.u)}{2c^2}\right] \quad (4)$$

$$g_0^{eq} = \omega_i\rho T\left[1 - \frac{3(u.u)}{2c^2}\right] \quad (5)$$

$$g_{1-4}^{eq} = \omega_i\rho T\left[1 + \frac{3(c_i.u)}{c^2} + \frac{9(c_i.u)^2}{2c^4} - \frac{3(u.u)}{2c^2}\right] \quad (6)$$

$$g_{5-8}^{eq} = \omega_i\rho T\left[1 + \frac{3(c_i.u)}{c^2} + \frac{9(c_i.u)^2}{2c^4} - \frac{3(u.u)}{2c^2}\right] \quad (7)$$

Where the weight factors ω_i are $\omega_0 = 4/9$ $\omega_{1-4} = 1/9$, $\omega_{5-8} = 1/36$ and c is the lattice speed and given by :

$$c = \Delta x/\Delta t = \Delta y/\Delta t \quad (8)$$

In lattice Boltzmann Method Equation (1) and (2) are solved in two important steps that are called collision and streaming step. Once the distribution functions are computed the macroscopic variable such as flow density ρ, velocity u and temperature T can be calculated in terms of these variables, with the following equations.

$$\rho u = \sum_i f_i c_i, \quad (9)$$

$$\rho = \sum_i f_i \,, \qquad (10)$$

$$T = \sum_i g_i \,, \qquad (11)$$

The corresponding kinematic viscosity ν and thermal diffusivity α are given by:

$$\nu = (\tau_f - 0.5)c_s^2 \Delta t \,, \qquad (12)$$

$$\alpha = (\tau_g - 0.5)c_s^2 \Delta t \text{ [14]}. \qquad (13)$$

Where $c_s^2 = c^2/3$ is the speed of sound.

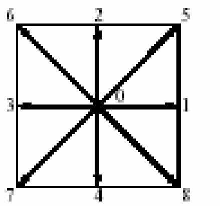

Figure.1. D_2Q_9 lattice model

To apply the LBM for micro flows, special treatment should be considered for the relaxation time τ which is related to the Knudsen number. From the kinetic theory the kinematic viscosity is $\nu = 0.5\bar{c}\lambda$ and the mean molecule velocity is $\bar{c} = \sqrt{8RT/\pi}$ combined with the kinematic viscosity expression of LB method we can have a simplified relation for the D_2Q_9 (Fig.1) lattice BGK model which is given by [13] :

$$Kn = \sqrt{\frac{\pi}{6}} \times \frac{\tau}{H\Delta t} \qquad (14)$$

3. BOUNDARY CONDITIONS

Boundary conditions play a crucial role in micro-geometries. In this work flow with inlet constant velocity is considered. A uniform lattice with a mesh of 41 × 164 nodes with aspect ratio of 4 is used to perform all of the simulations. Following the work of Zou and He [20], the inlet density and the distribution functions in the D_2Q_9 model can be obtained from:

$$\rho_{in} = \frac{1}{1 - u_{in}}\left[f_0 + f_2 + f_4 + 2(f_3 + f_6 + f_7)\right] \qquad (15)$$

The unknown distributions are calculated as:

$$f_1 = f_3 + \frac{2}{3}\rho_{in}u_{in} \qquad (16)$$

$$f_5 = f_7 - \frac{1}{2}(f_2 - f_4) + \frac{1}{6}\rho_{in}u_{in} \qquad (17)$$

$$f_8 = f_6 + \frac{1}{2}(f_2 - f_4) + \frac{1}{6}\rho_{in}u_{in} \qquad (18)$$

In some problems, the outlet velocity is not known. The normal practice is to use extrapolation for the unknown distribution functions.

Slip boundary conditions are used to specify the boundary conditions on solid surfaces, following the Maxwell first order slip boundary condition, the following slip (jump) boundary condition is used [13], [19].

$$u^{slip} = \sigma k_n \left(\frac{\partial u}{\partial y}\right)_w \qquad (19)$$

$$T^{jump} = \varphi \left(\frac{2\gamma}{\gamma + 1}\right)\left(\frac{k_n}{P_r}\right)\left(\frac{\partial T}{\partial y}\right)_w \qquad (20)$$

Where γ is the specific heat ratio, p_r is Pandtl number, $\sigma = (2 - \sigma_v)/\sigma_v$ and $\varphi = (2 - \sigma_T)/\sigma_T$ The tangential momentum accommodation coefficient σ_v is defined as the fraction of molecules reflected diffusively and σ_T is the thermal accommodation coefficient. These two coefficient are equal to 1 usually [19]. The first derivative of velocity and temperature at the wall of equations (19) and (20) are computed by adopting the second- order implicit scheme [19]. The slip boundary condition for bottom wall is as follow:

$$\rho_w = f_0 + f_1 + f_3 + 2(f_7 + f_4 + f_8) \qquad (21)$$

$$f_2 = f_4 \qquad (22)$$

$$f_5 = \frac{\rho_w(1 + u_x) - (f_0 + f_2 + f_4)}{2} - (f_1 + f_8) \qquad (23)$$

$$f_6 = \frac{\rho_w(1 - u_x) - (f_0 + f_2 + f_4)}{2} - (f_3 + f_7) \qquad (24)$$

$$u_x = k_n \frac{(4u_{x,1} - u_{x,2})}{2 + 3k_n} \qquad (25)$$

Where $u_{x,1}$ and $u_{x,2}$ are the first two velocity values after the fluid velocity on the wall.
Temperature boundary condition is implemented using the equality of the non equilibrium distribution functions [1].The equation set below is derived for inlet boundary.

$$g_1 = T_{in}\left(\omega(1)+\omega(3)\right) - g_3 \tag{26}$$

$$g_5 = T_{in}\left(\omega(5)+\omega(7)\right) - g_7 \tag{27}$$

$$g_8 = T_{in}\left(\omega(8)+\omega(6)\right) - g_6 \tag{28}$$

The outlet boundary condition for temperature can be implemented by using an extrapolation scheme [1]. Similar to velocity derivative the spatial derivative of temperature in equation (20) is also calculated using a second-order implicit scheme, hence equations of temperature jump for bottom wall are:

$$T_{y=0} = \frac{\left[C\left(4T_1 - T_2\right) + 2T_w\right]}{\left(2+3C\right)} \tag{29}$$

$$g_2 = T_{y=0}\left(\omega(2)+\omega(4)\right) - g_4 \tag{30}$$

$$g_5 = T_{y=0}\left(\omega(5)+\omega(7)\right) - g_7 \tag{31}$$

$$g_6 = T_{y=0}\left(\omega(6)+\omega(8)\right) - g_8 \tag{32}$$

Where C is a temperature jump coefficient defined as:

$$C = f k_n \left(\frac{2\gamma}{(\gamma+1)p_r}\right) = k \times k_n \tag{33}$$

Where :

$$k = \frac{2\gamma}{(\gamma+1)p_r} \tag{34}$$

4. ANALYTICAL SOLUTION

Velocity component in the x direction can be obtained by x-component of the linear momentum equation which is supported by the Maxwell boundary conditions at walls. This equation is a parabolic curve. When a non-dimensional length is defined as $\eta = Y/H$ the solution is

$$\frac{U}{U_{mean}} = -\frac{6\left(\eta^2 - \eta - k_n\right)}{6k_n + 1} \tag{35}$$

5. NUMERICAL COMPUTATION

In the present simulation we studied forced convection inside a microchannel. The flow is driven between 2 parallel plates (Fig.2), The characteristic length scales of this problem are the channel dimensions H (thickness) and L (length) where both non-dimensional temperature at the inlet and at the walls are 1 and 0 respectively, the plates are crossed by a laminar flow of forced convection. Constant thermal properties of fluid flow are considered. We have studied the effect of governing parameters such as, Knudsen number, Prandtl number on velocity and temperature profiles and on Nusselt number. Boundary velocity and temperature values depend on Knudsen number, therefore we repeated the simulation for several Knudsen numbers of 0.01, 0.05, 0.08 covering the slip regime.

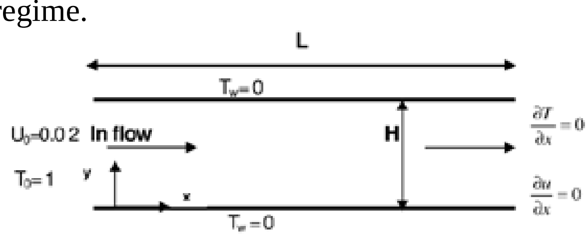

Figure.2. Geometrical configuration.

6. RESULTS AND DISCUSSIONS

To observe the validity of the boundary condition implementation, first the velocity profile for different Knudsen numbers are plotted together with the analytical solution, where perfect agreement is observed (Fig.3). This figure shows the effect of Knudsen number on the slip velocity and on the velocity at the center line of the microchannel. On the other hand (Fig.4) shows the effect of Knudsen number on the temperature jump and on the temperature profile for a fixed Prandtl number of 0.71 which is a typical value of air, the wall temperature jump increases with increasing *kn*. With the fixed *kn* and *k* given by equation (34), (Fig.5) shows the temperature profile variation when *Pr number* is increased. Nusselt number depends on the velocity and temperature profiles, convergence of Nusselt number to a constant value is reached as soon as velocity and temperature become fully developed. The value of Nusselt number indicates the ratio of heat transfer due to convection and conduction in the microchannel.

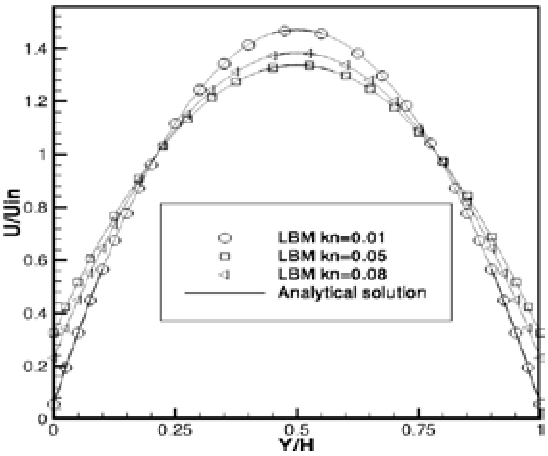

Figure.3.Comparaison of LBM and analytical solution of velocity for different Knudsen numbers. *Re*=10.

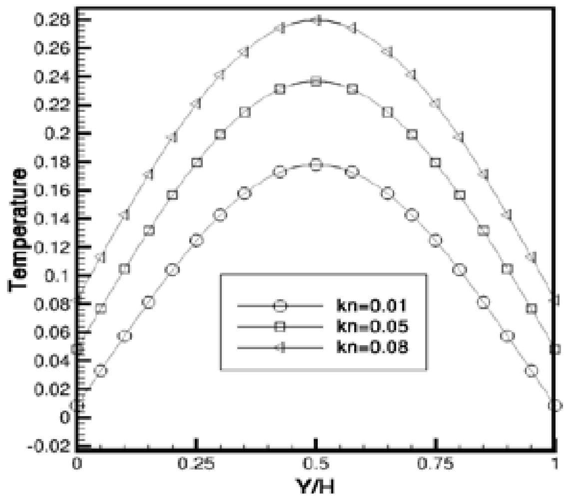

Figure.4.LBM temperature profile for different Knudsen number.*Pr*=0.71, *k*=1.667, *Re*=10.

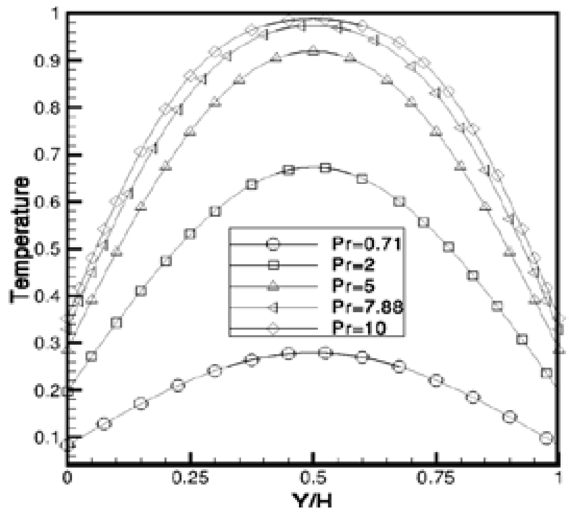

Figure.5.LBM temperature profile for different Prandtl number *Re*=10, *kn*=0.08, *k*=1.667.

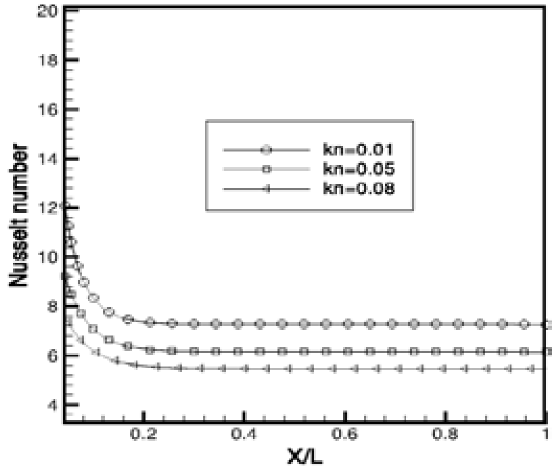

Figure.6.Nusselt number variation along the microchannel for different Knudsen number *Re*=10, *Pr*=0.71, *k*=1.667.

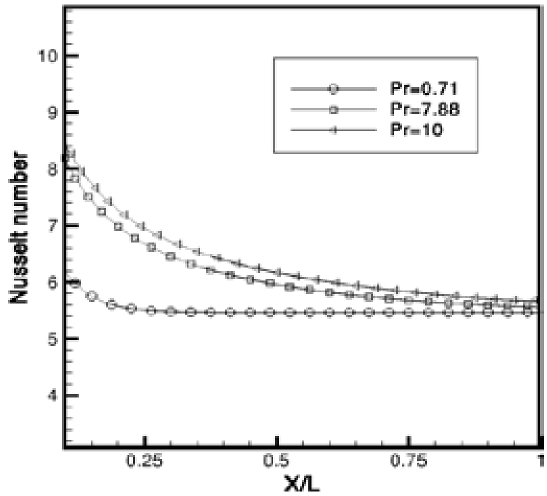

Figure.7.Nusselt number variation along the microchannel for different Pr number *Re*=10, *Kn*=0.08.

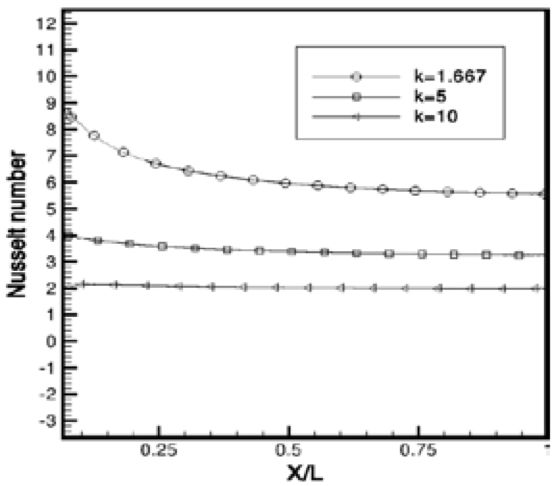

Figure.8.Nusselt number variation along the microchannel for different values of parameter *k*, *Re*=10, *Kn*=0.08, *Pr*=7.88.

(Fig.6) shows Nusselt number variation along the microchannel for developed regimes for *k*=1.667 and for *kn*=0.01,0.05,0.08, increasing Knudsen number leads to decreased Nusselt number. It has the maximum value at the start of entrance region then decreases gradually until it will be close to thermal fully developed region. In (Fig.7) Nusselt number variation along the microchannel for different Pr number of 0.71 which is a typical value of air and 7.88 a typical value of water are used, as we can see, increasing Pr number leads to increased Nusselt number. (Fig.8) provides the variation of Nusselt number along the microchannel with different values of the parameter *k.* For *k*=10 which corresponds to a large temperature jump case, the variation of Nusselt number along the microchannel is much smaller than for *k*=5 and *k*=1.667 and Nusselt number has lower values. So the conduction rate increases with *k* and *kn* which resulted in decrease in Nusselt number.

7.CONCLUSIONS

The (TLBM) simulation for fluid flow through the microchannel was performed , the velocity slip and the temperature jump of the wall boundary were adopted. Results were found for the velocity profile along the micro channel for different Knudsen number, these results are plotted together with the analytical solution and good matches are observed. We studied the effect of governing parameters such as Knudsen number, Prandtl number and the parameter *k* on the velocity profile, temperature profile and on the Nusselt number. From the present results, conduction rate increases with *k* and *kn* which resulted in decrease in Nusselt number, this means that increasing the Knudsen number of the flow makes the flow to approach the transition regime where heat transfer by convection loses its importance and conduction becomes more significant due to the rarefaction effects. The present study reveals many interesting features of the microchannel flow with heat transfer. More investigations are to be considered by using more sophisticated (LB) models such as (MRT) and (TRT) models.

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